1st Question:

\[ P = P_s \left(1 + \frac{\rho}{100}\right)^{t-t_s} \]

Taking logarithms of both sides:

\[ \log P = (\log P_s) + (t-t_s) \log \left(1 + \frac{\rho}{100}\right) \]

Then replacing \( \frac{\rho}{100} = \left[ \frac{P}{P_t} - \frac{1}{(t-t_s)} \right] - 1 \):

\[ \log P = (\log P_s) + (t-t_s) \left[ \frac{1}{t-t_s} \log \frac{P_s}{P_t} \right] \]

After transforming, we finally get the same formula:

\[ \log P = (\log P_s) + \frac{(\log P_s) - (\log P_t)}{(t-t_s)}(t-t_s) \]

2nd Question:

a) Taking logarithms of both sides:

\[ \log P = \log P_s + k_s (t-t_s) \log e \]

\( e \) is the natural logarithm base. As we know:

\[ k_s = \frac{(\log P_s) - (\log P_t)}{(t-t_s)} \frac{1}{\log e} \]

this formula can be used for geometrical extrapolation.

b) The formulae in Q1 and Q2 is compared:

\[ P_s \left(1 + \frac{\rho}{100}\right)^{t-t_s} = P_s e^{k_s(t-t_s)} \]

so \( (1 + \frac{\rho}{100}) = e^{k_s} \).

When we take \( \ln() \) of both sides; \( k_s = \ln(1 + \frac{\rho}{100}) \)
**3rd Question:**

a) \( P_{1995} = P_{1975} e^{k_e (1985-1975)} \) from here; 

\[ k_g = (\ln \frac{100000}{90000}) \frac{1}{10} = 0.010536. \]

\[ P_{1995} = P_{1985} e^{k_e (1995-1985)} = 100000 \times e^{0.0105 \times 10} = 111111 \]

---

b) 

![Graph showing population growth](image)

The slope of the population vs. time curve at 1995 is 

\[ \frac{dy}{dt} = k_g P_{95} = k_g \times 111111 \]

when calculated from the past side.

The same slope is:

\[ \frac{dy}{dt} = k_d (L - P_{95}) = k_d \times (300000 - 111111) = 188889k_d \]

when calculated from the future side.

As we should consider a common slope at this point:

\[ 188889k_d = 111111k_g \]

From here 

\[ k_d = \frac{111111}{188889} k_g = 0.589k_g = 0.006205. \]

\[ L - P_{2005} = (L - P_{1995}) e^{-k_e (2005-1995)} \]

\[ 300000 - P_{2005} = (300000 - 111111) e^{-0.006205 \times (2005-1995)} = 177525 \]

\[ P_{2005} = 122475 \]