CEVAPLAR

1. Consider Gary’s utility function: \( U(X,Y) = 5XY \), where \( X \) and \( Y \) are two goods. If the individual consumed 10 units of \( X \) and received 250 units of utility, how many units of \( Y \) must the individual consume? Would a market basket of \( X=15 \) and \( Y=3 \) be preferred to the above combination? Explain.

\[ \text{Solution:} \]

Given that \( U(X,Y) = 5XY = 5(10)Y \), then \( 250 = 50Y \), or \( Y = 5 \).

Since this individual receives 250 units of satisfaction with \( (X = 10, Y = 5) \), would \( (Y = 3 \) and \( X = 15) \) be a preferred combination? At these values, \( U = 5(15)(3) = 225 \). So, the first combination would be preferred.

2. An island economy produces only two goods, coconuts and pineapples. There are five people (A, B, C, D, and E) living on the island with these preferences:
   - A has a strong preference for pineapples.
   - B has a strong preference for coconuts.
   - C doesn’t care for pineapples (assigns no value to them).
   - D doesn’t care for coconuts (assigns no value to them)
   - E will only consume pineapples and coconuts in the fixed proportion of one pineapple to one coconut.

For each of these five individuals, construct a representative indifference curve with pineapples on the vertical axis and coconuts on the horizontal axis. Discuss the shape of the indifference curves and relate them to the MRS.

\[ \text{Solution:} \]

For each individual, construct a representative indifference curve with pineapples on the vertical axis and coconuts on the horizontal axis. The shape of the indifference curves will reflect the preferences of each individual, and the MRS will indicate the marginal rate of substitution.
Individual A has relatively flat indifference curves, since A requires relatively large numbers of coconuts to compensate for the loss of pineapples that she values highly.

Individual B has relatively steep indifference curves, since B requires relatively few coconuts to compensate for the loss of pineapples that he does not value highly.

C's indifference curves are vertical; the level of satisfaction is affected only by coconuts.

D's indifference curves are horizontal; the level of satisfaction is affected only by pineapples.

E's indifference curves are L-shaped.

MRS measures (at the margin) the maximum number of pineapples that the consumer will be willing to give up in order to get one more unit of coconuts.

A’s MRS is low. A is willing to relinquish few pineapples relative to coconuts since pineapples are dear to A. B’s MRS is high, for the opposite reason.

C’s MRS is infinite. Since C’s utility is not affected by pineapples, she is willing to relinquish all pineapples (an infinite number) to obtain additional coconuts.

D’s MRS is zero. D is not willing to give up any pineapples to obtain additional coconuts.

E’s MRS is infinite when $Q_p > Q_c$, zero when $Q_c > Q_p$, and undefined when $Q_c = Q_p$.

3. Sally consumes two goods, X and Y. Her utility function is given by the expression $U = 2XY^3$. The current market price for X is $10, while the market price for Y is $5.00. Sally's current income is $500.

a. Write the expression for Sally's budget constraint. Graph the budget constraint and determine its slope.

b. Determine the X,Y combination which maximizes Sally's utility, given her budget constraint. Show her optimum point on a graph. (Partial quantities are possible.)

(Note: $MU_Y = 6XY$ and $MU_X = 3Y^2$.)

c. Calculate the impact on Sally's optimum market basket of an increase in the price of X to 15. What would happen to her utility as a result of the price increase?

Solution:

a. $I = P_xX + P_yY$

500 = 10X + 5Y

Slope = $\frac{-100}{50} = -2$
b. To maximize utility, Sally must find the point where

MRS is equal to \( \frac{P_X}{P_Y} \).

\[
MRS = \frac{MU_X}{MU_Y}
\]

recall: \( MU_Y = 6XY \), \( MU_X = 3Y^2 \)

\[
MRS = \frac{3Y^2}{6XY} = \frac{Y}{2X}
\]

\[
\frac{P_X}{P_Y} = \frac{10}{5} = 2
\]

set \( MRS = \frac{P_X}{P_Y} \)

\[
\frac{Y}{2X} = 2
\]

\( Y = 4X \)

Sally should consume four times as much \( Y \) as \( X \).

To determine exact quantities, substitute \( Y = 4X \) into

\[
I = P_X X + P_Y Y
\]

\[
500 = 10X + 5Y
\]

\[
500 = 10X + 5(4X)
\]

\[
500 = 30X
\]

\( X = 16.67 \)

\( Y = 4(16.67) \)

\( Y = 66.67 \)

c.

\[
MRS \text{ remains } \frac{Y}{2X}, \quad \frac{P_X}{P_Y} \text{ becomes } \frac{15}{5} = 3
\]

Equating \( MRS \) to \( \frac{P_X}{P_Y}, \frac{Y}{2X} = 3, Y = 6X \)

Substitute \( Y = 6X \) into the equation

\[
500 = 15X + 5Y
\]

\[
500 = 15X + 5(6X)
\]

\[
500 = 45X
\]

\( X = 11.11 \)

\( Y = 6(11.11) \)

\( Y = 66.67 \)

Before price change:

\[
U = 3(16.67)(66.67)^2 = 222,289.
\]
After price change:

\[ U = 3(11.11)(66.67)^2 = 148,148. \]

Utility fell due to the price change. Sally is on a lower indifference curve. (Note: Answers may be slightly different due to rounding)

4. Janice Doe consumes two goods, X and Y. Janice has a utility function given by the expression:

\[ U = 4X^{0.5}Y^{0.5}. \]

The current prices of X and Y are 25 and 50, respectively. Janice currently has an income of 750 per time period.

a. Write an expression for Janice’s budget constraint.

\[ I = PxX + PyY \]

\[ 750 = 25X + 50Y \]

b. Calculate the optimal quantities of X and Y that Janice should choose, given her budget constraint. Graph your answer.

Optimal Combination:

\[ MRS = \frac{P_x}{P_y} \]

\[ MRS = \frac{MU_x}{MU_y} = \frac{2}{2} \cdot \frac{Y^5}{X^5} \]

\[ MRS = \frac{Y}{X} \]

\[ \frac{P_x}{P_y} = \frac{25}{50} = \frac{1}{2} \]

Equating MRS to \( \frac{P_x}{P_y} \):

\[ \frac{Y}{X} = \frac{1}{2} \]

Janice should buy 1/2 as much Y as X.

Recall 750 = 25X + 50Y

Substitute \( (1/2)X \) for Y

\[ 750 = 25X + 50(1/2)X \]
\[750 = 25X + 25X\]
\[750 = 50X\]
\[X = 15\]
\[Y = (1/2)X\]
\[Y = (1/2)(15)\]
\[Y = 7.5\]

Janice should consume 7.5 units of Y and 15 units of X.

\[750 = 25X + 50Y\]
\[X = 10\]
\[750 = 25(10) + 50Y\]
\[500 = 50Y\]
\[Y = 10\]

As indicated in the graph below, at Janice’s optimal bundle with the restriction, \(\frac{MU_X}{P_X} > \frac{MU_Y}{P_Y}\). This implies Janice should consume more X to increase utility. However, the ration restriction prevents her from doing so. Given the restriction, this is the best Janice can do.
d. Janice's utility without the restriction is: \( U(x = 15, y = 7.5) = 4(15)^{0.5}(7.5)^{0.5} = 42.43. \)

Janice's utility with the restriction is: \( U(x = 10, y = 10) = 4(10)^{0.5}(10)^{0.5} = 40. \) The ration restriction results in a utility loss of 2.43 utils for Janice.

5. Lisa's budget line and her satisfaction maximizing market basket, A, are shown in the diagram below.

\[ \text{Food} \]
\[ \text{Other Goods} \]

- A

a. Suppose that Lisa is given $50 worth of coupons that must be spent on food. How will the coupons alter Lisa's budget line?
b. Suppose that Lisa is given $50 in cash instead of $50 in coupons. How will this alter Lisa's budget line?
c. Is Lisa indifferent between the food coupon and cash program, or does she prefer one program over the other? Draw an indifference curve to illustrate your answer.

**Solution:**

Refer to the following diagram with the answers.

\[ \text{Food} \]
\[ \text{Other Goods} \]

- d
- b
- a
- c

- a. With the coupons, Lisa’s budget is abc.
- b. With cash, Lisa’s budget line is dbc.
- c. If Lisa’s preferences are as shown, she is indifferent between the two programs. However, if her preferences were such that an indifference curve was tangent to the db portion of dbc, she would prefer cash.