1. Complete the following table (round each answer to the nearest whole number):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>5</td>
<td>30</td>
<td>35</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>30</td>
<td>30</td>
<td>25</td>
<td>30</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>110</td>
<td>80</td>
<td>30</td>
<td>50</td>
<td>37</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>170</td>
<td>30</td>
<td>90</td>
<td>50</td>
<td>43</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>320</td>
<td>290</td>
<td>30</td>
<td>120</td>
<td>64</td>
<td>58</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>600</td>
<td>570</td>
<td>30</td>
<td>280</td>
<td>100</td>
<td>95</td>
<td>5</td>
</tr>
</tbody>
</table>

2. A firm's total cost function is given by the equation:

\[ TC = 4000 + 5Q + 10Q^2. \]

(1) Write an expression for each of the following cost concepts:

a. Total Fixed Cost
b. Average Fixed Cost
c. Total Variable Cost
d. Average Variable Cost
e. Average Total Cost
f. Marginal Cost

(2) Determine the quantity that minimizes average total cost. Demonstrate that the predicted relationship between marginal cost and average cost holds.

Solution:

**PART (1)**

a. 

\[ TFC = 4000 \]
b. 
\[ AFC = \frac{4000}{Q} \]

c. 
\[ TVC = TC - TFC \]
\[ TVC = 5Q + 10Q^2 \]

d. 
\[ AVC = \frac{TVC}{Q} = \frac{5Q + 10Q^2}{Q} = 5 + 10Q \]

e. 
\[ ATC = \frac{TC}{Q} = \frac{4000 + 5Q + 10Q^2}{Q} \]

f. 
\[ MC = 5 + 20Q \]
PART (2)

ATC is minimized where MC is equal to ATC.

Equating MC to ATC

\[
\frac{4000 + 5Q + 10Q^2}{Q} = 5 + 20Q
\]

\[
4000 + 5Q + 10Q^2 = 5Q + 20Q^2
\]

\[
4000 = 10Q^2
\]

\[
Q^2 = 400
\]

\[
Q = 20
\]
ATC is minimized at 20 units of output. Up to 20, ATC falls, while beyond 20 ATC rises. MC should be less than ATC for any quantity less than 20.

For example, let Q = 10:

\[
MC = 5 + 20(10) = 205
\]

\[
ATC = \frac{4000 + 5(10) + 10(10)^2}{10} = 505
\]

MC is indeed less than ATC for quantities smaller than 20.

MC should exceed ATC for any quantity greater than 20.

For example, let Q = 25:

\[
MC = 5 + 20(25) = 505
\]

\[
ATC = \frac{4000 + 5(25) + 10(25)^2}{25} = 415
\]

MC is indeed greater than ATC for quantities greater than 20.

3. Acme Container Corporation produces egg cartons that are sold to egg distributors. Acme has estimated this production function for its egg carton division:

\[
Q = 25L^{0.6}K^{0.4}
\]

where \( Q \) = output measured in one thousand carton lots,

\( L \) = labor measured in person hours, and \( K \) = capital measured in machine hours. Acme currently pays a wage of $10 per hour and considers the relevant rental price for capital to be $25 per hour. Determine the optimal capital-labor ratio that Acme should use in the egg carton division.

**Solution:**

\[
MP_L = .6(25)L^{-0.4}K^{0.4} = 15\frac{K^{0.4}}{L^{0.4}}
\]

\[
MP_K = .4(25)L^{0.6}K^{-0.6} = 10\frac{L^{0.6}}{K^{0.6}}
\]

\[
\text{MRTS} = \frac{MP_L}{MP_K}
\]

\[
\text{MRTS} = \frac{15\frac{K^{0.4}}{L^{0.4}}}{10\frac{L^{0.6}}{K^{0.6}}} = 1.5\frac{K^{0.4}}{L^{0.4}} \cdot \frac{L^{0.6}}{K^{0.6}}
\]

\[
\text{MRTS} = 1.5\frac{K}{L}
\]

Equate MRTS to \( \frac{w}{r} \):

\[
1.5\frac{K}{L} = \frac{10}{25}
\]

\[
1.5\frac{K}{L} = 0.4
\]

\[
1.5K = 0.4L; \quad K = 0.266L
\]
4. Davy Metal Company produces brass fittings. Davy’s engineers estimate the production function represented below as relevant for their long-run capital-labor decisions.

\[ Q = 500L^{0.6}K^{0.8}, \]

where \( Q \) = annual output measured in pounds,

\( L \) = labor measured in person hours,

\( K \) = capital measured in machine hours.

The marginal products of labor and capital are:

\[ MP_L = 300L^{-0.4}K^{0.8} \]

\[ MP_K = 400L^{0.6}K^{-0.2} \]

Davy’s employees are relatively highly skilled and earn $15 per hour. The firm estimates a rental charge of $50 per hour on capital. Davy forecasts annual costs of $500,000 per year, measured in real dollars.

a. Determine the firm’s optimal capital-labor ratio, given the information above.

b. How much capital and labor should the firm employ, given the $500,000 budget? Calculate the firm’s output.

c. Davy is currently negotiating with a newly organized union. The firm’s personnel manager indicates that the wage may rise to $22.50 under the proposed union contract. Analyze the effect of the higher union wage on the optimal capital-labor ratio and the firm’s employment of capital and labor. What will happen to the firm’s output?

**Solution:**

\[ a. \]

\[ MP_L = 300L^{-0.4}K^{0.8} = 300 \frac{K^{0.8}}{L^{0.4}} \]

\[ MP_K = 400L^{0.6}K^{-0.2} = 400 \frac{L^{0.6}}{K^{0.2}} \]

\[ MRTS = \frac{300 \frac{K^{0.8}}{L^{0.4}}}{400 \frac{L^{0.6}}{K^{0.2}}} = 0.75 \frac{K^{0.8}}{L^{0.4}} \cdot \frac{L^{0.6}}{K^{0.2}} \]

\[ MRTS = 0.75 \frac{K}{L} \]

Equate to \( \frac{w}{r} = \frac{15}{50} \)

\[ 0.75 \frac{K}{L} = \frac{15}{50} \]

\[ 0.75 \frac{K}{L} = 0.3 \]

\[ \frac{K}{L} = 0.4; K = 0.4L \]
\[ b. \]

\[ C = 500,000 \]
\[ C = wL + rK \]
\[ 500,000 = 15L + 50K \]

K = 0.4L from optimal ratio

\[ 500,000 = 15L + 50(0.4L) \]
\[ 500,000 = 15L + 20L \]
\[ 500,000 = 35L \]
\[ L = 14,285.71 \text{ or } 14,286 \text{ hours} \]

Substitute to solve for K.

\[ 500,000 = 15(14286) + 50K \]
\[ 500,000 = 214,290 + 50K \]
\[ 285,710 = 50K \]
\[ K = 5714.20 \]
\[ \text{or } K = 5714 \]

\[ Q = 500(14,286)^{0.6}(5,714)^{0.8} \]
\[ Q = 157,568,191 \]

c.

\[ \text{MRTS} = 0.75 \frac{K}{L} \]

New \[ \frac{w}{r} = \frac{22.5}{50} = 0.45 \]

Equating MRTS to \[ \frac{w}{r} = \frac{22.5}{50}. \]

\[ 0.75 \frac{K}{L} = 0.45 \]
\[ \frac{K}{L} = 0.6 \]

K = 0.6L

Substitute into C:

\[ 500,000 = 22.50L + 50K \]
\[ K = 0.60L \]
\[ 500,000 = 22.50L + 50(0.6L) \]
\[ 500,000 = 22.50L + 30L \]
\[ 500,000 = 52.50L \]
\[ L = 9,523.8 \text{ or } 9,524 \]

L fell from 14,286 to 9,524. Substitute to solve for K.

\[ 500,000 = 22.50(9,524) + 50K \]
\[ 285,710 = 50K \]
\[ K = 5,714.20 \text{ or } 5,714 \]

K remains constant.
Output fell from 157,568,202.5 to 123,541,771.8.

5.

a. Homer's boat manufacturing plant leases 50 hydraulic lifts and produces 25 boats per period. Homer's short-run cost function is: \( C(q, K) = 15 \frac{q^5}{K^{7/2}} + 200K \), where \( q \) is the number of boats produced and \( K \) is the number of hydraulic lifts. Homer's long-run cost function is: \( C_{LR}(q) = 173.5578q^{10/7} \). At Homer's current short-run plant size, calculate Homer's short-run average total cost of production. If Homer would lease 11 more hydraulic lifts in the short run, will his short-run average total cost of producing 25 boats increase or decrease? Does Homer's long-run cost function exhibit increasing, constant, or decreasing returns to scale?

**Solution:** At Homer's current short-run plant size, Homer's short-run average total cost of production is:

\[
ATC(25,50) = \frac{15(25)^5}{(50)^{7/2}} + 200(50) = 731.46.
\]

If Homer leases an additional 11 hydraulic lifts, short-run average total costs become:

\[
ATC(25,61) = \frac{15(25)^5}{(61)^{7/2}} + 200(61) = 689.62.
\]

We see that Homer's short-run average total costs decrease if he uses 11 additional hydraulic lifts.

Homer's long-run average costs are:

\[
AC_{LR}(q) = \frac{C_{LR}(q)}{q} = \frac{173.5578q^{10/7}}{q} = 173.5578q^{3/7}.
\]

Since long-run average costs increase as output increases, Homer's production process has decreasing returns to scale.

b. Marge's Hair Salon uses 15 hair dryers to produce 10 units of output per period. Marge's short-run cost function is: \( C(q, K) = \frac{15q^2}{K} + 12K \), where \( q \) is the number of units produced and \( K \) is the number of hair dryers Marge leases. Marge's long-run cost function is: \( C_{LR}(q) = 26.8q \). If Marge used 4 fewer hair dryers in the short-run, would short-run average total costs increase or decrease? Does Marge's long-run cost curve exhibit increasing, constant, or decreasing returns to scale?

**Solution:** Currently, Marge's short-run average costs are:

\[
SRATC(10,15) = \frac{\left(\frac{15(10)^2}{15} + 12(15)\right)}{10} = 28.00.
\]

If Marge uses 4 fewer hair dryers in the short run, her short-run average total costs become:

\[
SRATC(10,11) = \frac{\left(\frac{15(10)^2}{11} + 12(11)\right)}{10} = 26.84.
\]

If Marge uses 4 fewer dryers and produces 10 units, here short-run average total costs decrease. Marge's long-run average costs are:

\[
LRAC = \frac{C_{LR}(q)}{q} = \frac{26.8q}{q} = 26.8.
\]

We see that Marge's long-run average costs are constant. This implies that Marge's cost curve exhibit constant returns to scale.
c. Apu leases 2 squishy machines to produce 40 squishies in the short run. Apu’s short-run cost function is: $C(q,K) = 0.85 \frac{q^2}{K^2} + 0.5K$, where $q$ is the number of squishies produced and $K$ is the number of squishy machines used. Apu’s long-run cost function is: $C_{LR}(q) = 1.13q^{\frac{2}{3}}$. If Apu decides to lease 7 squishy machines, what happens to Apu’s short-run average total cost of producing 40 squishies? Does Apu’s long-run cost function exhibit increasing, constant, or decreasing returns to scale?

**Solution:**
With 2 squishy machines, Apu’s short-run average total costs are:

$$SRATC(40, 2) = \frac{0.85 \left( \frac{40^2}{2} \right) + 0.5(2)}{40} = 8.525.$$ 
If Apu leases 7 squishy machines, his short-run average total costs become: $$SRATC(40, 7) = \frac{0.85 \left( \frac{40^2}{7} \right) + 0.5(7)}{40} = 0.78.$$ 
Leasing 5 additional squishy machines lowers Apu’s short-run average total cost by 91%. Apu’s long-run average cost curve is: $LRAC(q) = \frac{1.13q^{\frac{2}{3}}}{q^{\frac{1}{3}}} = \frac{1.13}{q^{\frac{1}{3}}}$. Since Apu’s long-run average costs decrease as output increases, Apu’s cost curve exhibit increasing returns to scale.

6. The cost of producing 600 small fiberglass sailboats per year, and the cost of producing sails and fittings necessary to make the boats seaworthy in a single plant, are together $780,000. If produced in separate plants, the boats would cost $540,000, and the sails and fittings would cost $180,000. From this information, what can be learned about (1) economies of scale and (2) economies of scope in the production of sailboats, sails, and fittings? Perform any necessary calculations and explain.

**Solution:**
The above information says nothing about economies of scale. However, one can calculate the degree of economies of scope. Use equation (7.7).

$$SC = \frac{C(Q_1) + C(Q_2) - C(Q_1, Q_2)}{C(Q_1, Q_2)} \frac{540,000 + 180,000 - 780,000}{780,000} = -0.077$$
Since SC is negative but close to zero, there are slight diseconomies of scope.

7. LeAnn’s Telecommunication firm long-run cost curve is: $C(q) = 16 \frac{q^A}{A^q}$, where $q$ is the number of units produced and $A$ is the time in months that LeAnn’s manager has spent on the job. What happens to production costs as the manager gains more experience on the job? Is this experience-effect common in production processes?

**Solution:**
Increases in $A$ will reduce the average cost of production for any quantity. This implies that as the manager gains job experience, LeAnn’s cost of production will decrease.
Suppose that LeAnn is producing 16 units and the manager has 1 month experience. LeAnn's costs are: \( C(16) = 16 \frac{(16)^{\sqrt{4}}}{(1)^{\sqrt{6}}} = 128 \). If LeAnn's manager has 256 months of experience, LeAnn's costs are: \( C(16) = 16 \frac{(16)^{\sqrt{4}}}{(256)^{\sqrt{6}}} = 16 \).

This experience-effect of the manager is referred to in economics as a "learning-curve" effect. Many production processes exhibit a learning-curve effect. That is, more time spent performing an activity results in greater efficiency and smaller costs.

8. Murray Manufacturing Company produces pantyhose. The firm's production function is given as:

\[ Q = 5LK, \]

where \( Q \) = pairs of pantyhose, \( L \) = labor measured in person hours, and \( K \) = capital measured in machine hours. Murray's labor cost, including fringe benefits, is $20 per hour, while the firm uses $80 per hour as an implicit machine rental charge per hour. Murray's current budget is $64,000 per month to pay labor and capital.

a. Given the information above, determine Murray's optimal capital/labor ratio.
b. Using the Lagrangian technique, determine the quantities of labor and capital that will allow the firm to maximize output given their budgeted input expenditure. What is the firm's output?
c. Again using the Lagrangian technique, demonstrate the duality in production and cost theory.

**Solution:**

a. optimal capital/labor ratio

\[
\begin{align*}
\frac{MP_L}{MP_K} &= \frac{w}{r} \\
MP_L &= \frac{\partial Q}{\partial L} = 5K \\
MP_K &= \frac{\partial Q}{\partial K} = 5L \\
\frac{MP_L}{MP_K} &= \frac{5K}{5L} = \frac{K}{L} \\
\text{Equating } \frac{MP_L}{MP_K} &= \frac{w}{r},
\end{align*}
\]

\[
\begin{align*}
K &= \frac{20}{80} \\
L &= \frac{1}{4} \\
K &= \frac{L}{4}
\end{align*}
\]
b.

Lagrangian is to maximize Q subject to cost constraint.

\[
\text{Max } Q = 5LK
\]

subject to

\[
64,000 = 20L + 80K
\]

Form the Lagrangian function

\[
G = 5LK + \lambda (64,000 - 20L - 80K)
\]

First order conditions are:

\[
\begin{align*}
(1) \frac{\partial G}{\partial L} &= 5K - 20\lambda = 0 \\
(2) \frac{\partial G}{\partial K} &= 5L - 80\lambda = 0 \\
(3) \frac{\partial G}{\partial \lambda} &= 64,000 - 20L - 80K = 0
\end{align*}
\]

Solve (1) and (2) to eliminate \( \lambda \).

\[
\begin{align*}
5K - 20\lambda &= 0 \\
5L - 80\lambda &= 0
\end{align*}
\]

\[
\Rightarrow
\begin{align*}
20K - 80\lambda &= 0 \\
5L - 80\lambda &= 0 \\
20K - 5L &= 0
\end{align*}
\]

Solve with expression for \( \frac{\partial G}{\partial \lambda} \).

\[
\begin{align*}
64,000 - 20L - 80K &= 0 \\
-5L + 20K &= 0
\end{align*}
\]

\[
\begin{align*}
64,000 - 20L - 80K &= 0 \\
-20L + 80K &= 0
\end{align*}
\]

\[
\begin{align*}
64,000 - 40L &= 0 \\
64,000 &= 40L \\
L &= 1600 \\
-5L + 20K &= 0 \\
-5(1600) + 20K &= 0 \\
-8000 + 20K &= 0 \\
20K &= 8000 \\
K &= 400
\end{align*}
\]
L = 1600, K = 400
Q = 0.5(1600)(400)
Q = 3,200,000

c.
To demonstrate duality one must show that cost minimization approach leads to same answer as maximizing quantity.

Minimize \( C = 20L + 80K \)
subject to
\( 3,200,000 = 5LK \)

Form Lagrangian function:
\[
G^* = 20L + 80K + \lambda(3,200,000 - 5LK)
\]

First Order Conditions are:
\[
\begin{align*}
(1) \quad \frac{\partial G^*}{\partial L} &= 20 - 5\lambda K = 0 \\
(2) \quad \frac{\partial G^*}{\partial K} &= 80 - 5\lambda L = 0 \\
(3) \quad \frac{\partial G^*}{\partial \lambda} &= 3,200,000 - 5LK = 0 
\end{align*}
\]

Solve 1 and 2 to eliminate \( L \).
\[
\begin{align*}
20 - 5\lambda K &= 0 \\
80 - 5\lambda K &= 0 \\
\frac{20}{K} - 5\lambda &= 0 \\
\frac{80}{L} - 5\lambda &= 0 \\
20 / K - 80 / L &= 0
\end{align*}
\]

Combine with 3 to solve for \( L \) and \( K \).
\[
\begin{align*}
20 / K - 80 / L &= 0 \\
3,200,000 - 5LK &= 0
\end{align*}
\]
Multiply top equation by \(L^2K\).

\[
\begin{align*}
20L^2 - 80LK &= 0 \\
3,200,000 - 5LK &= 0
\end{align*}
\]

\[
\begin{align*}
20L^2 - 80LK &= 0 \\
51,200,000 - 80LK &= 0 \\
20L^2 - 51,200,000 &= 0
\end{align*}
\]

\[
\begin{align*}
20L^2 - 51,200,000 &= 0 \\
20L^2 &= 51,200,000 \\
L^2 &= 2,560,000 \\
L &= 1600
\end{align*}
\]

\[
\begin{align*}
3,200,000 - 5(1600)K &= 0 \\
-8000K &= -3,200,000 \\
K &= 400
\end{align*}
\]

We find the identical \(K\) and \(L\) as with output maximization approach.