25. In the diagram below, Marvin’s optimal consumption bundles are indicated for five different budget constraints. Sketch the Engel curve for Marvin. Next, use the diagram to sketch Marvin’s demand curve for the good on the horizontal axis.

Solution: To construct the Engel curve, the relevant budget constraints are 3, 4 and 5. The Engel curve will be increasing in income and quantity space. To construct the demand curve, the relevant budget constraints are 1, 2 and 3. Demand for good 1 increases as the price decreases.

26. Donald derives utility from only two goods, carrots (Qc) and donuts (Qd). His utility function is as follows:
\[ U(Qc, Qd) = (Qc)(Qd) \]

The marginal utility that Donald receives from carrots (MUc) and donuts (MUd) are given as follows:
\[ MUc = Qd \quad MUd = Qc \]

Donald has an income (I) of $120 and the price of carrots (Pc) and donuts (Pd) are both $1.

a. What is Donald’s budget line?
b. What is Donald’s income-consumption curve?
c. What quantities of Qc and Qd will maximize Donald’s utility?
d. Holding Donald’s income and Pd constant at $120 and $1 respectively, what is Donald’s demand curve for carrots?
e. Suppose that a tax of $1 per unit is levied on donuts. How will this alter Donald’s utility maximizing market basket of goods?
f. Suppose that, instead of the per unit tax in (e), a lump sum tax of the same dollar amount is levied on Donald. What is Donald’s utility maximizing market basket?
g. The taxes in (e) and (f) both collect exactly the same amount of revenue for the government, which of the two taxes would Donald prefer? Show your answer numerically and explain why Donald prefers the per unit tax over the lump sum tax, or vice versa, or why he is indifferent between the two taxes.
Solution:

a. Budget line: \( 120 = Q_c + Q_d \)

b. The income consumption curve must satisfy: 
   \[ \frac{MU_d}{MU_c} = \frac{P_d}{P_c} \]
   Substituting for \( MU_d, MU_c, P_d, \) and \( P_c \) yields: 
   \[ \frac{Q_c}{Q_d} = 1 \text{ or } Q_c = Q_d \]

c. Substituting the information in (b) into the budget line: 
   \[ 120 = Q_c + Q_c = 2Q_c \]
   \[ Q_c = 60 \]
   \[ Q_d = 60 \]

d. Rewriting the budget line: 
   \[ 120 = P_cQ_c + Q_d \]
   Substituting the information in (b) into the budget line: 
   \[ 120 = P_cQ_c + Q_c = Q_c(P_c + 1) \]
   \[ Q_c = \frac{120}{(P_c + 1)} \]

e. The $1 tax on donuts raises the after-tax price to $2. The income-consumption curve becomes: 
   \[ \frac{MU_d}{MU_c} = \frac{P_d}{P_c} \]
   Substituting for \( MU_d, MU_c, P_d \) and \( P_c \) yields: 
   \[ \frac{Q_c}{Q_d} = 2 \text{ or } Q_c = 2Q_d \]
   The budget line is: 
   \[ 120 = Q_c + 2Q_d \]
   Substitute the income-consumption curve into the budget line to eliminate \( Q_c \): 
   \[ 120 = 2Q_d + 2Q = 4Q_d \]
   \[ Q_d = 30 \]
   \[ Q_c = 60 \]
f. Donald buys 30 donuts, so he pays $30 in tax. If Donald paid $30 in a lump sum tax, his income would be $90. Resolve the utility maximization problem with \( I = 90, \ P_c = P_d = 1 \).

The utility maximizing market basket is \( Q_c = Q_d = 45 \).

g. Donald prefers the lump-sum tax to the excise tax. Use the utility function to show which market basket is preferred.

\[
U(Q_c, Q_d) = Q_c Q_d
\]

\[
\text{lump-sum tax:} \quad U(45, 45) = 45 \times 45 = 2,025
\]

\[
\text{excise tax:} \quad U(60, 30) = 60 \times 30 = 1,800
\]

27. The demand curves for steak, eggs, and hot dogs are given in the table below. The current price of steak is $5. The price of eggs is $2.50, and the price of hot dogs is $0.75. Fill in the remaining columns of the table using this information. Indicate which goods are substitutes and which goods are complements.

<table>
<thead>
<tr>
<th>Good</th>
<th>Demand Equation</th>
<th>Steak Price Elasticity of Demand</th>
<th>Egg Price Elasticity of Demand</th>
<th>Hotdog Price Elasticity of Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steak</td>
<td>( D_s = 500 - 2P_s - \frac{1}{10}P_e + \frac{1}{100}P_h )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Egg</td>
<td>( D_e = 75 - 3P_e - P_s + \frac{1}{10}P_h )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hotdog</td>
<td>( D_h = 300 - \frac{1}{2}P_h + P_s + \frac{1}{10}P_e )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Solution:**

<table>
<thead>
<tr>
<th>Good</th>
<th>Demand Equation</th>
<th>Steak Price Elasticity of Demand</th>
<th>Egg Price Elasticity of Demand</th>
<th>Hotdog Price Elasticity of Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steak</td>
<td>( D_s = 500 - 2P_s - \frac{1}{10}P_e + \frac{1}{100}P_h )</td>
<td>-0.020</td>
<td>-0.00051</td>
<td>1.53</td>
</tr>
<tr>
<td>Egg</td>
<td>( D_e = 75 - 3P_e - P_s + \frac{1}{10}P_h )</td>
<td>-0.079</td>
<td>-0.24</td>
<td>1.20E-4</td>
</tr>
<tr>
<td>Hotdog</td>
<td>( D_h = 300 - \frac{1}{2}P_h + P_s + \frac{1}{10}P_e )</td>
<td>0.016</td>
<td>0.00082</td>
<td>-0.0012</td>
</tr>
</tbody>
</table>

Steak and eggs are complements. Steak and hotdogs and eggs and hotdogs are substitutes.
28. Answer both parts of the following question.

a. The San Francisco Chronicle reported that the toll on the Golden Gate Bridge was raised from $2 to $3. Following the toll increase, traffic fell by 5 percent. Based on this information, calculate the point price elasticity of demand. Is demand elastic or inelastic? Explain.

b. Stephen Leonoudakis, chairman of the bridge's finance auditing committee, warned that the toll increase could cause toll revenues to decrease by $2.8 million per year. Is this statement consistent with economic theory? Explain.

Solution:

a.
Increasing the toll on the bridge from $2 to $3 is a 50 percent increase. Traffic is expected to decrease by 5 percent as a result of the toll increase. Therefore, the point price elasticity of demand is -5/50 or -0.1. Demand is inelastic.

b.
Stephen Leonoudakis' statement is not consistent with economic theory. When demand is inelastic, an increase in price will increase total expenditures on a good (the total expenditure on the good is the total revenue of the firm). Since demand is inelastic here, toll revenues will increase rather than decrease.

29. The wheat market is perfectly competitive, and the market supply and demand curves are given by the following equations:

\[ QD = 20,000,000 - 4,000,000P \]
\[ QS = 7,000,000 + 2,500,000P, \]

where QD and QS are quantity demanded and quantity supplied measured in bushels, and \( P \) = price per bushel.

a. Determine consumer surplus at the equilibrium price and quantity.

b. Assume that the government has imposed a price floor at $2.25 per bushel and agrees to buy any resulting excess supply. How many bushels of wheat will the government be forced to buy? Determine consumer surplus with the price floor.

Solution:

a.
The first step is to determine the equilibrium price \( (P_e) \) and quantity \( (Q_e) \) by equating QD and QS.

\[ 20,000,000 - 4,000,000P_e = 7,000,000 + 2,500,000P_e \]
\[ 13,000,000 = 6,500,000P_e \]
\[ P_e = $2.00. \]

Substitute into QD or QS

\[ Q_e = 20,000,000 - 4,000,000(2) \]
\[ Q_e = 12,000,000. \]

To find consumer surplus, we must also determine the choke price \( (P_c) \). That is, the price at which quantity demanded is zero. Solve for \( P \) in terms of QD and QS.

\[ 0 = 20,000,000 - 4,000,000P_c \]
At price of 2.25
\[
\begin{align*}
Q_D &= 20,000,000 - 4,000,000(2.25) \\
Q_D &= 11,000,000 \\
Q_S &= 7,000,000 + 2,500,000(2.25) \\
Q_S &= 12,625,000
\end{align*}
\]
Excess supply is \(Q_S - Q_D\).
\[
12,625,000 - 11,000,000 = 1,625,000
\]
Government should expect to buy 1,625,000 bushels.

\[
Cs = (0.5)(P_c-2.25)Q_D = (0.5)(5-2.25)(11,000,000) = 15,125,000.
\]
C.S. fell from 18,000,000 to 15,125,000.
30. The following table gives the current price, quantity, and price elasticities of the linear demand curves for pencils, paper and scissors. The columns $E_{rc}$ under the Price Elasticities heading are calculated as $E_{rc} = \left( \frac{\Delta Q_r}{\Delta P_c} \right) \frac{P_c}{Q_r}$. The terms $r$ and $c$ refer to the row of the table and the column under the price elasticities heading, respectively. For example, if $r$ is one and $c$ is two, the value $E_{12}$ is the responsiveness of pencil demand to changes in the paper price (i.e., a cross-price elasticity). The demand curves for each good are in the form $Q_r = a_r + b_r P_1 + c_r P_2 + d_r P_3$. Using the information in the table, derive the demand curve for each good.

<table>
<thead>
<tr>
<th>Demand Item</th>
<th>Own Price</th>
<th>Quantity</th>
<th>Price Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$E_{r1}$</td>
</tr>
<tr>
<td>Pencils</td>
<td>0.35</td>
<td>25,000</td>
<td>-1.2</td>
</tr>
<tr>
<td>Paper</td>
<td>2.00</td>
<td>90,000</td>
<td>0.01</td>
</tr>
<tr>
<td>Scissors</td>
<td>3.15</td>
<td>1,500</td>
<td>0</td>
</tr>
</tbody>
</table>

**Solution:** Using the information in the table allows us to solve for the coefficients on prices as follows: $b_r = E_{r1} \left( \frac{Q_r}{P_1} \right)$; $c_r = E_{r2} \left( \frac{Q_r}{P_2} \right)$; and $d_r = E_{r3} \left( \frac{Q_r}{P_3} \right)$. Substituting these coefficient values into the demand equation allows us to solve for $a_r$. This is done by setting $a_r = Q_r - (b_r P_1 + c_r P_2 + d_r P_3)$. Performing these calculations for the first row of the table gives us the demand for pencils $Q_1 = 48,750 - 85,714.3 P_1 + 3,125 P_2 + 0 P_3$. The demand for paper is $Q_2 = 125,000 + 2,571.43 P_1 - 38,250 P_2 + 12,857.14 P_3$. The demand for scissors is $Q_3 = 2,325 + 0 P_1 + 900 P_2 - 833.33 P_3$. 