1) Two rigid tanks (A and B, as shown in figure) connected to each other by a valve contain air at specified conditions. Initially the valve is closed and the state of air in each tank is as shown in figure. Later the valve is opened letting the pressure in tanks A and B equalise and the system reach thermal equilibrium with the surroundings of 20°C Temperature.

a) Assuming ideal gas, determine the volume of tank B, and the final equilibrium pressure of air (air in now united tanks A and B).

b) Determine how appropriate the assumption of ideal gas is for the given states of air.

3-80 Two rigid tanks connected by a valve to each other contain air at specified conditions. The volume of the second tank and the final equilibrium pressure when the valve is opened are to be determined.

**Assumptions** At specified conditions, air behaves as an ideal gas.

**Properties** The gas constant of air is \( R = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K} \) (Table A-1).

**Analysis** Let's call the first and the second tanks A and B. Treating air as an ideal gas, the volume of the second tank and the mass of air in the first tank are determined to be

\[
V_B = \left( \frac{m_B R T_A}{P} \right) = \left( \frac{(5 \text{ kg})(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(308 \text{ K})}{200 \text{ kPa}} \right) = 2.21 \text{ m}^3
\]

\[
m_A = \left( \frac{R V}{RT_A} \right)_A = \left( \frac{(500 \text{ kPa})(1.0 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} \right) = 5.846 \text{ kg}
\]

Thus,

\[
V = V_A + V_B = 1.0 + 2.21 = 3.21 \text{ m}^3
\]

\[
m = m_A + m_B = 5.846 + 5.0 = 10.846 \text{ kg}
\]

Then the final equilibrium pressure becomes

\[
P = \frac{m R T}{V} = \left( \frac{(10.846 \text{ kg})(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(308 \text{ K})}{3.21 \text{ m}^3} \right) = 284.1 \text{ kPa}
\]
2) A rigid vessel with 10 l volume initially contains a mixture of liquid water and water vapour at temperature of 100°C with 12.3 % quality. The vessel is slowly heated until its temperature increases to 150°C.
   a) Determine the internal energy (u), quality (x), specific volume (v), pressure (P) for initial and final states.
   b) Show the process on the P-v diagram in relation to vapour dome (saturated liquid and saturated vapour curves). You are required to label the states, show pressure and specific volume values, indicate appropriate lines of constant temperature.
   c) Determine the heat transfer during the heating process.

Problem 2: (30 points)

Given:
A rigid vessel with volume 10 L initially contains a mixture of liquid water and vapor at temperature of 100°C with 12.3% quality (state 1). The vessel is slowly heated until its temperature increases to 150°C (state 2).

Find:
(a) Complete the following table for water. Write any necessary assumptions, equations, and show your solution in the spaces provided. (11 points)

<table>
<thead>
<tr>
<th>State</th>
<th>Temperature (°C)</th>
<th>Specific Volume (m³/kg)</th>
<th>Specific Internal Energy (kJ/kg)</th>
<th>Phase</th>
<th>Quality (%) (If Applicable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.2067</td>
<td>675.71</td>
<td>SLVM</td>
<td>12.3%</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>0.2067</td>
<td>1643.79</td>
<td>SLVM</td>
<td>52.5%</td>
</tr>
</tbody>
</table>

(b) Show the process from state 1 to state 2 on the P-v diagram in relation to the vapor dome. You must label the states, show pressure and specific volume values, and indicate appropriate lines of constant temperature. (8 points)

(c) Determine the heat transfer (kJ) required during the heating process. Write any necessary assumptions, equations, and show your solution in the spaces provided. (4 points)

(c) Heat transfer required during the heating process = +46.86 kJ
Assumptions: (2 points)  
- Rigid tank is closed system  
- $\Delta KE = 0$, $\Delta PE = 0$  
- No work interactions for the system

Basic Equations: (5 points)  
\[
y = y_f + x \left( y_g - y_f \right)
\]
\[
m = \frac{V}{v}
\]
\[
Q - W = \Delta U + \Delta KE + \Delta PE \implies Q_{12} = U_2 - U_1 = m \left( u_2 - u_1 \right)
\]

Solution:
(a) State 1: $T_1 = 100^\circ C$, $x_1 = 0.123$

Using Table A-2: $v_i = v_f + x_i \left( v_g - v_f \right) = 0.0010435 \frac{m^3}{kg} + 0.123 \times (1.673 - 0.0010435) \frac{m^3}{kg}$

$\implies v_1 = 0.2067 \frac{m^3}{kg}$

Similarly, $u_i = u_f + x_i \left( u_g - u_f \right) = 418.94 \frac{kJ}{kg} + 0.123 \times (2506.5 - 418.94) \frac{kJ}{kg}$  \[\implies u_1 = 675.71 \frac{kJ}{kg}\]

State 2: $v_2 = v_1$ (rigid tank $\implies$ total volume and total mass constant i.e. specific volume same)  
Using Table A-2: $v_f$ at 150$^\circ$C $< v_2 = 0.2067 \frac{m^3}{kg} < v_g$ at 150$^\circ$C $\implies$ SLVM  

Quality at state 2: $x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.2067 - 0.0010905}{0.3928 - 0.0010905} \implies x_2 = 0.525$

$u_2 = u_f + x_2 \left( u_g - u_f \right) = 631.68 \frac{kJ}{kg} + 0.525 \times (2559.5 - 631.68) \frac{kJ}{kg}$  \[\implies u_2 = 1643.79 \frac{kJ}{kg}\]

(b) See the P-v diagram above.

(c) Considering energy balance, the heat transfer during the heating process is: $Q_{12} = m \left( u_2 - u_1 \right)$

Mass inside the rigid tank: $m = \frac{V}{v_j} = \frac{(10 / 1000) \frac{m^3}{1}}{0.2067 \frac{m^3}{kg}} = 0.0484 \text{ kg}$

i.e. $Q_{12} = 0.0484 \text{ kg} \times (1643.79 - 675.71) \frac{kJ}{kg}$ i.e. $Q_{12} = +46.86 \text{ kJ}$; positive sign indicates heat transfer into the system
3) A steady-flow compressor is used for compressing Argon (Ar) gas from 100 kPa and 20°C at the inlet to 1500 kPa and 320°C at the outlet. The velocity at the inlet is 15 m/s, while the velocity and area at the exit are 30 m/s and 95 cm² respectively.
   a) Calculate the mass flow rate of Argon through the compressor
   b) What is the area at the inlet of the compressor?

Solution:
(a) Using Table A-1 for Argon: $P_{\text{critical}} = 48.6$ bar and $T_{\text{critical}} = 151$ K
   At the inlet: Reduced pressure: $P_{\text{r1}} = \frac{P_1}{P_{\text{critical}}} = 0.0206$
   Reduced temperature: $T_{\text{r1}} = \frac{T_1}{T_{\text{critical}}} = \frac{(20 + 273) \text{K}}{151 \text{K}} = 1.94$

   Using Figure A-1: $Z_1 = 1 \Rightarrow$ ideal gas
   Similarly, at the exit: $P_{\text{r2}} = 0.3086$ and $T_{\text{r2}} = 3.93 \Rightarrow Z_2 = 1 \Rightarrow$ ideal gas

(b) Flow of Argon gas through the compressor is shown below.

Since Argon can be modeled as ideal gas, the specific volume of Argon at the exit of the compressor:

$$v_2 = \frac{R_{\text{Ar}} T_2}{P_2} = \frac{0.2082 \text{kJ} \text{kg-K}}{39.94 \text{kJ} \text{kg-kmol}} \times \frac{(320 + 273) \text{K}}{(15 \times 100) \text{kPa}} = 0.0823 \text{m}^3 \text{kg}^{-1}$$

Mass flow rate of Argon: $\dot{m} = \dot{m}_2 = \frac{A_2 v_2}{v_2} = \frac{(95 \times 10^{-4} \text{m}^2) \times 30 \text{m}}{0.0823 \text{m}^3 \text{kg}^{-1}} \Rightarrow \dot{m} = 3.46 \text{ kg s}^{-1}$

(c) Since Argon can be modeled as ideal gas, the specific volume of Argon at the inlet of the compressor:

$$v_1 = \frac{R_{\text{Ar}} T_1}{P_1} = \frac{0.2082 \text{kJ} \text{kg-K}}{(1 \times 100) \text{kPa}} \times (20 + 273) \text{K} = 0.61 \text{ m}^3 \text{kg}^{-1}$$

Considering mass balance for the compressor:

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{A_1 v_1}{v_1} = \frac{A_2 v_2}{v_2} \Rightarrow A_1 = A_2 \frac{v_2}{v_1} = 95 \text{ cm}^2 \times \frac{30 \text{ m}}{15 \text{ m}} \times \frac{0.61 \text{ m}^3 \text{kg}^{-1}}{0.0823 \text{ m}^3 \text{kg}^{-1}}$$

Area at the inlet of the compressor: $A_1 = 1408.3 \text{ cm}^2$

Exam duration is 90 minutes.
The marking of questions; Q1 is 30 points, Q2 and Q3 are 35 points each.
Good luck!