

## Neutron star solutions in perturbative quadratic gravity

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# Neutron star solutions in perturbative quadratic gravity

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**Abstract.** We study the structure of neutron stars in  $R + \beta R^{\mu\nu} R_{\mu\nu}$  gravity model with perturbative method. We obtain mass-radius relations for six representative equations of state (EoSs). We find that, for  $|\beta| \sim 10^{11} \text{ cm}^2$ , the results differ substantially from the results of general relativity. Some of the soft EoSs that are excluded within the framework of general relativity can be reconciled for certain values of  $\beta$  of this order with the 2 solar mass neutron star recently observed. For values of  $\beta$  greater than a few  $10^{11} \text{ cm}^2$  we find a new solution branch allowing highly massive neutron stars. By referring some recent observational constraints on the mass-radius relation we try to constrain the value of  $\beta$  for each EoS. The associated length scale  $\sqrt{\beta} \sim 10^6 \text{ cm}$  is of the order of the typical radius of neutron stars, the probe used in this test. This implies that the true value of  $\beta$  is most likely much smaller than  $10^{11} \text{ cm}^2$ .

**Keywords:** modified gravity, neutron stars

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## 1 Introduction

Einstein's theory of gravity, general relativity, enjoyed impressive observational and experimental support in the past century. Main successes of this theory were explanations it brought to phenomena such as precession of the perihelion of Mercury, bending of light and the gravitational redshift of light near massive bodies. Although all of these impressive tests were done solely in the solar system, the standard model of cosmology assumes the validity of general relativity in all scales up to the very large scale structure of the universe. However, as the 20th century was ending, data from distant supernovae Ia [1–3] were interpreted as evidence of late time acceleration in the expansion rate of the universe. If we continue to assume the validity of general relativity in all scales, then the best fit to observational data requires us to introduce a non-vanishing positive cosmological constant into the theory. This is the simplest way to follow without changing the basic paradigm. Yet there are several theoretical problems related with the existence of cosmological constant (see for example [4–7]). One of the most important among these problems is the lack of a quantum theoretical method to calculate its inferred value from cosmological data [8]. There are several proposals in order to avoid the problems of cosmological constant with alternative routes of explanations [9, 10]. These proposals can be collected into a few groups: to explain late time accelerated expansion one can either modify general relativity by modifying the Einstein-Hilbert action, or add new gravitational degrees of freedom other than the metric to the theory of gravity, or change how the matter fields and perhaps the cosmological constant gravitates [11, 12].

A modified gravity theory, which is proposed to solve late time cosmic acceleration problem, should also be able to pass several tests before it can be considered a viable theory of gravity. First of all, in the weak gravity regime, such a theory should be compatible with the solar system tests and table-top experiments. In cosmological scales, other than producing the late time accelerated expansion, it should be free of gravitational instabilities, and obey constraints of the standard model of cosmology. Such a theory is also expected to do well in strong gravity regime: for example it should have solutions for neutron stars with mass-radius relation inside the current observational bounds. There are alternatives to and generalizations of general relativity theory which have the same predictions in the weak-field regime as the general relativity, and also provide an explanation of the evolution of the universe in the large scale. Thus, the difference between general relativity and alternatives might become prominent in the strong-gravity regime [13].

One important family of modifications of Einstein-Hilbert action is the  $f(R)$  theories of gravity (see reviews [14–17] and references therein). In such theories one uses a function of curvature scalar as the Lagrangian density. The  $f(R)$  term must have a lower order expansion in Ricci scalar in order to include general relativity as, perhaps, a weak-field limit of it. Such models of gravity can be made to pass Solar System tests, explain the late-time accelerated expansion of the universe and also work well in the strong-field regime. In a previous work [18] a simple version of  $f(R)$  gravity theory, in which  $f(R) = R + \alpha R^2$ , is studied by two of the present authors. It is curious that predictions of such theories can be shown to be equivalent to scalar-tensor theories of gravity [19], which have been analyzed thoroughly since the seminal paper of Brans and Dicke [20]. However, if we modify the Einstein-Hilbert term by contractions of Ricci and Riemann tensors,  $R_{\mu\nu}R^{\mu\nu}$  and  $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ , then we have an alternative gravity theory with independent predictions.

Inspiration for such an alternative gravity theory may come from string theory. Absence of ghosts in low energy string theory in flat backgrounds requires the quadratic corrections to Einstein's gravity to be of the Gauss-Bonnet (GB) form [21]:

$$S = \int d^4x \sqrt{-g} [R + \gamma(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})] . \quad (1.1)$$

However, the Gauss-Bonnet term will not contribute to the equations of motion, because it is equivalent to a total derivative in four dimensions. This means that variations of  $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$  can be expressed in terms of variations of  $R^2$  and  $R_{\mu\nu}R^{\mu\nu}$ . Contraction of Riemann tensors, could have relevance only in the cases related to quantum gravity [22, 23]. Therefore for the classical physics applications one can take the action of our string-inspired gravity as

$$S = \int d^4x \sqrt{-g} [R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}] , \quad (1.2)$$

where  $\alpha$  and  $\beta$  are free-parameters, taken independent of each other. However, special relations between  $\alpha$  and  $\beta$  exists and such cases correspond to some special theories: for example,  $\beta = -3\alpha$  correspond to the Weyl tensor squared modification of general relativity [24, 25], and  $\beta = -4\alpha$  has unique energy properties [26].

As mentioned above, the absence of ghosts in the low energy string theory in flat backgrounds requires the quadratic corrections to Einstein's gravity to be of the form given in equation (1.1). This means that the alternative gravity theory defined by (1.2) cannot be free of ghosts and this brings up the question of stability of the neutron star solutions discussed in this paper. However, all such modified gravity theories, bar the theory defined with (1.1), suffer from the same problem, the solution of which is beyond the scope of this paper. Our aim in

this paper is to see the possibility of neutron star solutions in a specific modified gravity theory defined by (1.2) with  $\alpha = 0$ , to analyze solutions with mass-radius relations obeying the observational constraints, and to see if there are problems in this gravity model other than the well known stability issue. When combined with the results of [18] this analysis will allow for a comparison of the effects of  $R^2$  and  $R_{\mu\nu}R^{\mu\nu}$  terms in (1.2) on the structure of neutron stars.

We further note that the mass dimensions of quadratic corrections to Einstein-Hilbert action in (1.2) is order  $[L]^{-4}$ . It is possible to add terms with the same mass dimensions to the above action. In fact referring to the quantum gravity arguments in [23], we could also add the following dimension  $[L]^{-4}$  terms to the action:

$$\square R \quad \text{and} \quad \nabla^\mu \nabla^\nu R_{\mu\nu} . \quad (1.3)$$

However, as it is pointed out in [27] these terms do not contribute to the field equations and therefore in the context of the present paper they are also redundant.

The effect of the value of  $\alpha$ , while  $\beta = 0$ , on the mass-radius (M-R) relation of neutron stars has been studied, for a polytropic EoS in [28], and for realistic EoS in [18]. The latter work constrained the value of  $\alpha$  to be  $|\alpha| \lesssim 10^{10} \text{ cm}^2$ . In an other study, Santos analyzes neutron stars in this model of gravity for a single EoS of ideal neutron gas [29] and for a specific choice of  $\beta = -2\alpha$ ,  $\sqrt{\alpha} = 0.96 \text{ km}$ . In that work, the author shows, for this restricted choice of EoS and parameters, that stable configurations of neutron stars are possible even for arbitrarily large baryon numbers of the neutron star.

Our approach in this work is different than [29] in two ways: (i) Rather than choosing a fixed specific value for  $\beta$ , while  $\alpha = 0$ , we study its effect, as a free parameter, on the M-R relation and constrain its value by referring to observations, (ii) As the interaction between nucleons can not be neglected, the EoS of ideal neutron gas can not provide realistic M-R relations. We thus employ six different realistic EoSs corresponding to a variety of assumptions for the interaction between nucleons rather than the very restrictive EoS of ideal neutron gas.

In the present work, we adopt an alternative theory of gravity, in which the Einstein-Hilbert action is modified with the term  $R_{\mu\nu}R^{\mu\nu}$  only. In section 2, we assume a perturbative form of our alternative gravity model and obtain the field equations. The reason of perturbative approach is that the equations of motion derived from this alternative gravity theory are fourth order differential equations, and their treatment in four dimensions is problematic. To obtain the modified Tolman-Oppenheimer-Volkoff (TOV) equations from the field equations we also assumed perturbative forms of metric and hydrodynamical functions. In section 3, we solve the structure of neutron stars in this gravity model for six representative equations of state describing the dense matter of neutron stars. We plot the mass-radius relations of neutron stars for  $\beta$  changing in the range  $\sim \pm 10^{11} \text{ cm}^2$ . This way the value of the perturbative parameter  $\beta$  is constrained by the recent measurements of the mass-radius relation [30] and the observed 2 solar mass neutron star [31]. We identify that  $\beta \sim \pm 10^{11} \text{ cm}^2$  produces results that can have observational consequences. Lastly, the results of the numerical study and the significance of the scale of the perturbation parameter is discussed in the conclusions.

## 2 Modified TOV equations

The approach that we are going to take in this paper is in the spirit of [18], although not in details. The alternative theory that we are going to analyze is not the full quadratic gravity, but part of it whose analysis would complement the analysis reported in [18]. In the

appropriate units, the defining action of our alternative theory is given as

$$S = \int d^4x \sqrt{-g} (R + \beta R_{\mu\nu} R^{\mu\nu}) + S_{\text{matter}} , \quad (2.1)$$

where we already set  $G = 1$  and  $c = 1$ . This will also be the case in the rest of this section. We note that, in this study we are using the metric formalism of gravity in which matter only couples to the metric, and the Levi-Civita connection is a function of the metric. Variation of the action (2.1) with respect to the metric results in the field equations,

$$\begin{aligned} 8\pi T_{\mu\nu} = & G_{\mu\nu} + \beta \left( -\frac{1}{2} g_{\mu\nu} R_{ab} R^{ab} + \nabla^\rho \nabla_\rho R_{\mu\nu} \right) \\ & + \beta \left( -\nabla_\nu \nabla_\mu R - 2R_{\sigma\mu\nu\alpha} R^{\alpha\sigma} + \frac{1}{2} \square R g_{\mu\nu} \right) . \end{aligned} \quad (2.2)$$

where  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$  is the Einstein tensor. The energy-momentum tensor,  $T_{\mu\nu}$ , is the energy-momentum tensor of the perfect fluid.

In the second step we derive the hydrostatic equilibrium equations within the framework of the gravity model considered. The hydrostatic equilibrium equations, obtained and solved by Tolman-Oppenheimer and Volkoff [32, 33] within the framework of general relativity, are commonly called TOV equations. We use the same nomenclature in this paper, although the hydrostatic equilibrium equations in this gravity model will turn out to be quite different.

As in the case of general relativity we choose to work with a spherically symmetric diagonal form of the metric and metric functions depend only on the radial coordinate  $r$ :

$$ds^2 = -e^{2\phi_\beta} dt^2 + e^{2\lambda_\beta} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (2.3)$$

When this metric ansatz and the energy-momentum tensor for perfect fluid substituted into the field equations (2.2), the resulting equations will contain metric functions  $\phi(r)$  and  $\lambda(r)$ , hydrodynamic quantities  $P(r)$  and  $\rho(r)$ , as well as their derivatives with respect to  $r$ . The presence of these higher order derivatives precludes expressing the equation in terms of hydrodynamic quantities only and consequently deriving the modified TOV equations. In order to reach that aim we are going to use the perturbative approach [34, 35] where general relativistic solution is taken as the zeroth order solution of the field equations (2.2). As mentioned, this method had already been applied to  $f(R)$  models of gravity via perturbative constraints at cosmological scales [36, 37] and neutron stars with  $f(R) = R + \alpha R^{n+1}$  [18, 28]. In the perturbative approach,  $g_{\mu\nu}$  can be expanded perturbatively in terms of  $\beta$ :

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \beta g_{\mu\nu}^{(1)} + O(\beta^2) \quad (2.4)$$

Accordingly, the metric functions must also be expanded in terms of  $\beta$  as

$$\phi_\beta = \phi + \beta \phi_1 + \dots \quad \text{and} \quad \lambda_\beta = \lambda + \beta \lambda_1 + \dots , \quad (2.5)$$

and the hydrodynamic quantities of the perfect fluid are expanded perturbatively as:

$$\rho_\beta = \rho + \beta \rho_1 + \dots \quad \text{and} \quad P_\beta = P + \beta P_1 + \dots . \quad (2.6)$$

In these equations functions without subscript  $\beta$  are general relativistic solutions. In the present perturbative approach they are “zeroth order” solutions and obtained from Einstein equations,

$$-8\pi\rho = -r^{-2} + e^{-2\lambda}(1 - 2r\lambda')r^{-2}, \quad (2.7)$$

$$8\pi P = -r^{-2} + e^{-2\lambda}(1 + 2r\phi')r^{-2}, \quad (2.8)$$

In order to determine the first and the second TOV equations, we now evaluate  $tt$  and  $rr$  components of field equations (2.2) in terms of metric functions (2.3) and hydrodynamic quantities  $P(r)$  and  $\rho(r)$ . The  $tt$  component of the field equations turns out to be

$$8\pi\rho_\beta = \frac{1}{r^2}e^{-2\lambda_\beta} \left( 2r\lambda'_\beta - 1 + e^{2\lambda_\beta} \right) + \frac{1}{2}\beta (\rho^2 + 3P^2) + \frac{1}{2}\beta e^{-2\lambda} \left[ \begin{aligned} &\rho'' + 3P'' - R'' + \left( 2\lambda' + 2\phi' - \frac{1}{2}\frac{g'}{g} \right) R' \\ &+ \left( \phi' - \lambda' + \frac{2}{r} \right) (\rho' + 3P') - 4(\phi')^2 (\rho + P) \\ &- 2 \left( \phi' \lambda' - (\phi')^2 - \phi'' - \frac{2}{r}\phi' \right) (\rho - P) \end{aligned} \right], \quad (2.9)$$

and the  $rr$  component of the field equations is

$$8\pi P_\beta = \frac{1}{r^2}e^{-2\lambda_\beta} \left( 2r\phi'_\beta + 1 - e^{2\lambda_\beta} \right) - \frac{1}{2}\beta (\rho^2 + 3P^2) + \beta e^{-2\lambda} \left[ \begin{aligned} &P'' + \left( \phi' + \frac{2}{r} \right) \rho' - \left( 2\phi' + \lambda' + \frac{4}{r} \right) P' \\ &+ \left( \phi'' - \phi' \lambda' - (\phi')^2 + \frac{2}{r}\lambda' \right) \rho \\ &+ \left( 3\phi'' - 3\phi' \lambda' + (\phi')^2 - \frac{2}{r}\lambda' \right) P \end{aligned} \right]. \quad (2.10)$$

Here, prime denotes derivative with respect to  $r$ . Note that since terms multiplied with  $\beta$  are already first order in  $\beta$ , metric and hydrodynamic functions that appear in those terms are taken as zeroth order general relativistic functions. In all the other terms we have full functions. However, when we integrate TOV equations numerically, we are going to keep terms only up to first order in  $\beta$ .

We now define a mass parameter  $m_\beta$  by the relation  $e^{-2\lambda_\beta} = 1 - m_\beta/r$  so that the solution for the metric function  $\lambda_\beta(r)$  would have the same form of the exterior solution. Here, again we take  $m_\beta$  to be expanded in  $\beta$  as  $m_\beta = m + \beta m + \dots$  where  $m$  is the general relativistic zeroth order solution. It is given in terms of  $\rho(r)$  as

$$m = 8\pi \int \rho(r) r^2 dr \quad (2.11)$$

Taking the derivative of mass parameter,  $m_\beta$ , with respect to  $r$  we obtain

$$\frac{dm_\beta}{dr} = e^{-2\lambda_\beta} (2r\lambda'_\beta - 1 + e^{2\lambda_\beta}). \quad (2.12)$$

In the perturbative approach all terms which are multiplied with  $\beta$  in (2.9), (2.10) can be rearranged in terms of zeroth order general relativistic expressions by ignoring higher order terms. Using the expression (2.12) for  $\frac{dm_\beta}{dr}$  in  $tt$  field equation (2.9) we obtain the first modified TOV equation as

$$\frac{dm_\beta}{dr} = 8\pi\rho_\beta r^2 - \beta r^2 \left[ \begin{aligned} &3 \left( 1 - \frac{m}{r} \right) P'' - \frac{3}{2} \left( \rho r + 3\frac{m}{r^2} - \frac{4}{r} \right) P' + \frac{1}{2} \left( Pr + \frac{m}{r^2} \right) \rho' \\ &+ \rho^2 + P\rho - \frac{\rho+P}{2(r-m)} \left( P^2 r^3 + \frac{m^2}{r^3} + 2Pm \right) \end{aligned} \right]. \quad (2.13)$$

To derive the second modified TOV equation we use  $\nabla^\mu T_{\mu\nu} = 0$ , the conservation equation of energy-momentum tensor for a perfect fluid, which is equivalent to the hydrostatic equilibrium condition,

$$\frac{dP_\beta}{dr} = -(\rho_\beta + P_\beta) \frac{d\phi_\beta}{dr}. \quad (2.14)$$

We, therefore, need to read  $\frac{d\phi_\beta}{dr}$  from the  $rr$  field equation (2.10) and use in the above expression to obtain the second modified TOV equation. After some straightforward algebra we find it as

$$2(r - m_\beta) \frac{d\phi_\beta}{dr} = 8\pi r^2 P_\beta + \frac{m_\beta}{r} - \beta r^2 \left[ \begin{aligned} & \left(1 - \frac{m}{r}\right) P'' - \left(Pr + \frac{1}{2}\rho r - \frac{7}{2}\frac{m}{r^2} + \frac{4}{r}\right) P' \\ & + \left(\frac{1}{2}Pr + \frac{2}{r} - \frac{3}{2}\frac{m}{r^2}\right) \rho' \\ & \rho^2 + P\rho - 2\frac{m}{r^3}(\rho + P) \\ & - \frac{\rho + P}{2(r-m)} \left(P^2 r^3 + \frac{m^2}{r^3} + 2Pm\right) \end{aligned} \right]. \quad (2.15)$$

Note that one gets the original TOV equations in general relativity when one sets  $\beta$  to zero and transforms  $m \rightarrow 2m$ . The modified TOV equations, (2.13), (2.14) and (2.15), are complicated and therefore similar to the case in general relativity they are solved numerically. We explain the numerical analysis and present the results of it in the next section.

### 3 Solution of the neutron star structure

In this section we numerically solve the modified TOV equations, namely eqs. (2.13) and (2.14), with realistic equations of state (EoSs) appropriate for neutron stars.

#### 3.1 The equations solved

With the physical constants plugged and  $m \rightarrow 2m$  transformed, eqs. (2.13) and (2.14) become

$$\frac{dm}{dr} = 4\pi r^2 \rho + \frac{1}{2} \beta r^2 K \quad (3.1)$$

where

$$\begin{aligned} K = & - \left(1 + \frac{P}{\rho c^2}\right) \frac{G\rho^2}{c^2} - \left(1 - \frac{2Gm}{c^2 r}\right) 3 \frac{P''}{c^2} - \left(1 + \frac{2mc^2}{r^3 P}\right) \frac{GrP\rho'}{2c^4} \\ & + \frac{Gm}{rc^2} \frac{GP\rho}{2c^4} \left(1 + \frac{P}{\rho c^2}\right) \left(\frac{Pr^3}{mc^2} + \frac{4mc^2}{r^3 P} + 4\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1} \\ & + \left(\frac{\rho c^2}{P} + \frac{6mc^2}{r^3 P} - \frac{4c^4}{Gr^2 P}\right) \frac{3Gr^2 PP'}{2rc^6} \end{aligned} \quad (3.2)$$

and

$$\frac{dP}{dr} = - \frac{Gm\rho \left(1 + \frac{P}{\rho c^2}\right)}{r^2 \left(1 - \frac{2Gm}{c^2 r}\right)} \left(1 + \frac{4\pi r^3 P}{mc^2} + \frac{1}{2} \beta r^2 H\right) \quad (3.3)$$

where

$$\begin{aligned} H = & - \left(1 - \frac{2Gm}{rc^2}\right) \frac{rP''}{mc^2} - \left(\frac{1}{2} + \frac{2c^4}{Gr^2 P} - \frac{3mc^2}{r^3 P}\right) \frac{Gr^2 P\rho'}{mc^4} \\ & - \left(\frac{1}{2} \frac{\rho^2 c^4}{P^2} + \frac{1}{2} \frac{\rho c^2}{P} - \frac{2mc^2}{r^3 P} \frac{\rho c^2}{P} - \frac{2mc^2}{r^3 P}\right) \frac{2GrP^2}{mc^6} \\ & + \left(1 + \frac{1}{2} \frac{\rho c^2}{P} - \frac{7mc^2}{r^3 P} + \frac{4c^4}{Gr^2 P}\right) \frac{GP'r^2 P}{mc^6} \\ & + \left(\frac{1}{4} \frac{r^3 P}{mc^2} + \frac{mc^2}{r^3 P} + 1\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1} \left(1 + \frac{P}{\rho c^2}\right) \frac{P\rho}{c^2} \frac{2mG^2}{mc^4}. \end{aligned} \quad (3.4)$$



The dimension of the coupling constant  $\beta$  is length square. Therefore, perturbation is actually over the dimensionless quantity  $\bar{\beta} = \beta/\beta_0$ , where  $\beta_0$  is the nominal value appropriate for neutron stars. It is possible to define it in terms of fundamental constants as follows. The typical mass of degenerate stars, including neutron stars, in terms of fundamental constants is

$$M_0 = \frac{m_{\text{Pl}}^3}{m_{\text{n}}^2} = 1.8482M_{\odot} \quad (3.5)$$

where  $m_{\text{Pl}}$  is the Planck mass and  $m_{\text{n}}$  is the nucleon mass. Note that this can also be written as  $M_0 = \alpha_{\text{G}}^{-3/2}m_{\text{n}}$  where  $\alpha_{\text{G}} = Gm_{\text{n}}^2/\hbar c$  is the “gravitational coupling constant.” The radius of the neutron star which is about 10-15 km can be given in terms of this mass as three times its Schwarzschild radius. As  $\beta_0$  has the dimension of length square, it will be given as the square of this typical radius which is the only length scale in the system. Thus

$$\beta_0 = \left( \frac{6GM_0}{c^2} \right)^2 = 268.128 \text{ km}^2 \simeq 2.7 \times 10^{12} \text{ cm}^2. \quad (3.6)$$

Accordingly, we can expect that deviations from general relativity (GR), which come through the parameter  $\beta$ , become significant when  $\beta$  is a sizeable fraction of  $\beta_0$ . This is indeed what we observe through the numerical solution of the equations.

Another way to see the order of magnitude of coupling constant  $\beta$  is to see that a characteristic value of the Ricci scalar, and the square root of  $R^{\mu\nu}R_{\mu\nu}$ , is given by  $R_0 \sim GM_*/c^2 R_*^3 \sim G\rho_c/c^2$  where  $M_*$  is the mass and  $R_*$  is the radius of the star. As  $\beta_0 \sim 1/R_0$  we infer that  $\beta_0 \sim c^2/G\rho_c$ . In section 3.6 we check the validity of the perturbative approach by plotting the dimensionless coupling constant  $\beta/\beta_0$  for a range of static mass configurations.

In the above equations, the higher derivatives, like  $P'$ ,  $\rho'$  and  $P''$  are calculated by using the TOV equations obtained in GR. As such terms come only in the perturbative term, the error in employing GR instead of the full theory is of the order of  $\beta^2$ .

### 3.2 Equation of state

In order to solve the hydrostatic equilibrium equations (3.1) and (3.3) we must supplement them with an equation of state (EoS) defining the microscopic physics of neutron stars (NSs). The EoS of NS matter at the inner core where most of the mass resides is not well constrained. Different EoSs lead to different mass-radius (M-R) relations. As a result we have to solve the NS structure for a number of EoSs in order to show that our basic result, that  $|\beta| \lesssim 10^{12} \text{ cm}^2$ , does not depend on the EoSs. In this work we present results for six representative EoSs including typical EoS involving only nucleons as well as EoS involving nucleons and exotic matter. We do not make any analysis on strange quark stars as these are not gravitationally bound objects. We use an analytical representation of  $\log \rho(\log P)$  for all the EoSs obtained by fitting the tabulated EoS data following the method described in [38]. These EoSs are FPS [39], AP4 [40], SLy [41], MS1 [42], MPA1 [43] and GS1 [44]. The physical assumptions of these EoSs are described in [45].

### 3.3 Numerical method

We integrate eqs. (3.1) and (3.3) supplemented by an EoS numerically. We employ a Runge-Kutta scheme with fixed step size of  $\Delta r = 0.01 \text{ km}$  starting from the center of the star for a certain value of central pressure,  $P_c$ . We identify the radius of the star,  $R_*$ , as

the point where pressure drops to a very small value ( $10 \text{ dyne/cm}^2$ ) and record the mass contained inside this radial coordinate as the mass of the star,  $M_*$ .

We change the central pressure  $P_c$  from  $3 \times 10^{33} \text{ dyne cm}^{-2}$  to  $9 \times 10^{36} \text{ dyne cm}^{-2}$  in 200 logarithmically equal steps to obtain a sequence of equilibrium configurations. We record the mass and radius for each central pressure. This allows us to obtain a mass-radius (M-R) relation for a certain EoS. We then repeat this procedure for a range of  $\beta$  to see the effect of the perturbative term we added to the Lagrangian.

### 3.4 Observational constraints on the mass-radius relation

In order to constrain the value of  $\beta$  we have used the recent measurements of mass and radius of neutron stars, EXO 1745-248 [47], 4U 1608-52 [48] and 4U 1820-30 [49] (see [46] for a description of the method used). We use the  $2\sigma$  confidence contours, shown in all M-R plots as the region bounded by the thin black line, on the M-R relation of neutron stars given in [30], which is a union of these three constraints.<sup>1</sup>

Apart from the above we also use the mass of PSR J1614-2230 with  $1.97 \pm 0.04 M_\odot$  as recently measured by [31], as a constraint. It is shown as the horizontal black line with grey error-bar. For a viable combination of  $\beta$  and EoS, the maximum mass of the neutron star must exceed this measured mass.

These two constraints we employed exclude many of the possible EoSs within the framework of GR. The value of  $\beta$ , the coupling parameter of the gravity model studied in the present work, is a new degree of freedom which can alter the M-R relation if it takes values of order  $10^{11} \text{ cm}^2$ . By using this freedom one can determine the range of  $\beta$  values for each EoS that is consistent with the observational constraints.

### 3.5 The effect of $\beta$ on the M-R relation

We have determined the M-R relation for each EoSs for a range of  $\beta$  values. Results for each six representative EoSs are summarized below. In the following we use  $\beta_{11} \equiv \beta/10^{11} \text{ cm}^2$  instead of  $\beta$  as we are mostly interested with  $\beta$  of this order. For all EoS we observe that the maximum mass of a NS increases with decreasing value of  $\beta_{11}$  while the radius becomes smaller.

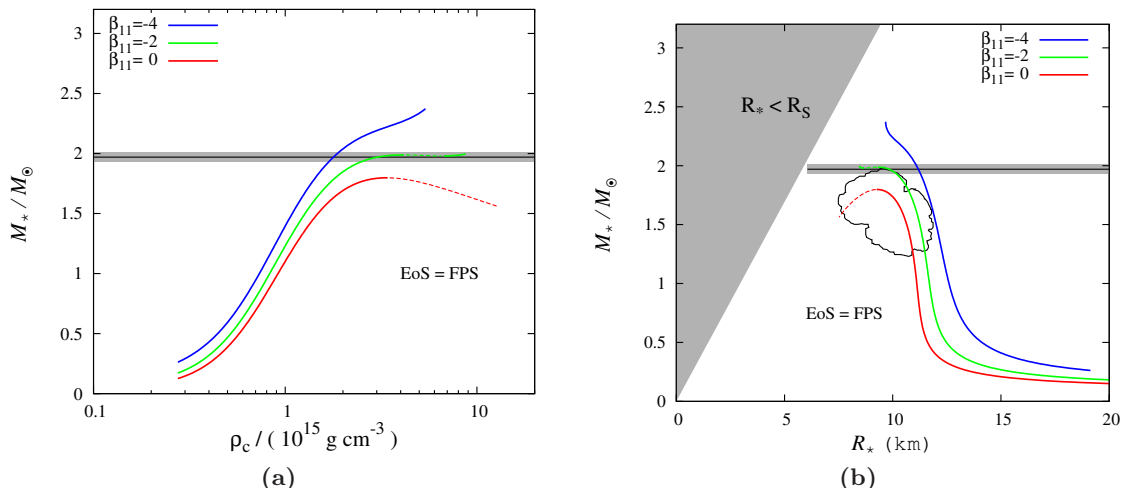
#### 3.5.1 FPS

The relation between the central density and the mass of the neutron star,  $\rho_c - M$ , and the M-R relation are shown in figure 1. For FPS, the maximum mass within GR, is about  $1.8 M_\odot$  which is less than the  $2M_\odot$  of PSR J1614-2230 meaning that FPS is excluded within GR ( $\beta = 0$ ).

From figure 1b we observe that the maximum stable mass of a NS increases with decreasing value of  $\beta$  and we find that, for  $\beta_{11} < -2$ , the maximum mass becomes  $M_{\text{max}} \simeq 2M_\odot$ . We thus find that FPS can be reconciled with the measurement of the maximum mass NS for  $\beta < -2 \times 10^{11} \text{ cm}^2$ .

For values of  $\beta_{11} < -4$  we see that the M-R relation does not pass through the confidence contours of the measured [30] mass and radius. This then implies that  $\beta_{11} > -4$ . Together with the previous constraint we conclude that FPS is consistent with observations if  $-4 < \beta_{11} < -2$  in this gravity model.

<sup>1</sup>See [50] for a critic of these constraints.



**Figure 1.** The  $\rho_c - M$  (panel a) and M-R (panel b) relation for FPS. The solid lines correspond to stable configurations for different values of  $\beta$ . The dashed lines correspond to the unstable configurations for which  $dM/d\rho_c < 0$ . The red line ( $\beta = 0$ ) stands for the results in GR. The thin black line on the right panel shows the  $2\sigma$  confidence contour of the observational measurements of [30]; the measured mass  $M = 1.97 \pm 0.04 M_\odot$  of PSR J1614-2230 [31] is shown as the horizontal black line with grey error bar. The grey shaded region shows where the radius of the NS would be less than the Schwarzschild radius. We observe that  $M_{\max}$  and  $R_{\min}$  increase for decreasing values of  $\beta$ . The comparison with observational constraints on the figure (see the discussion in the text) imply that  $-4 < \beta_{11} < -2$  in this gravity model if FPS is assumed to be the EoS of NSs.

Interestingly, we find that for  $\beta = -2 \times 10^{11} \text{ cm}^2$  there exists a new branch of stable solution at highest densities, in the sense that  $dM/d\rho_c > 0$ , which does not have a counterpart in GR.

For  $\beta_{11} = -3$  and lower values we find that, for the highest central densities,  $dm/dr < 0$  which indicates an unstable star. As a result such solutions are excluded from the figures 1a and 1b.

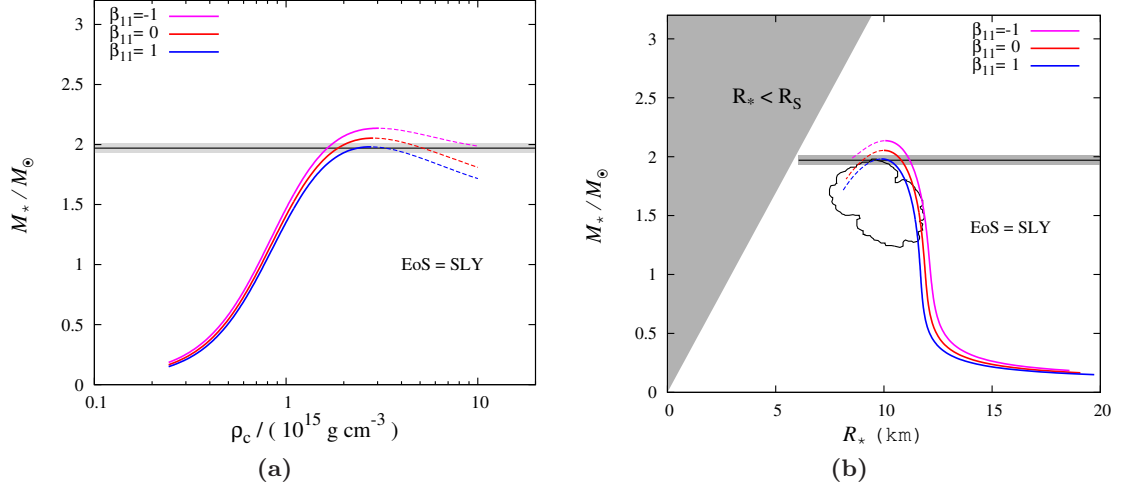
### 3.5.2 SLy

For SLy the  $\rho_c - M$  and M-R relation are shown in figure 2. For  $\beta_{11} > 2$  we see that  $M_{\max}$  is less than the measured mass of PSR J1614-2230 [31] and so is not compatible with its existence. For  $\beta_{11} < -1$ , the M-R relation does not pass through the confidence contours of [30]. These two constraints then imply that  $2 > \beta_{11} > -1$  for the gravity model employed here so that SLy is consistent with the observations.

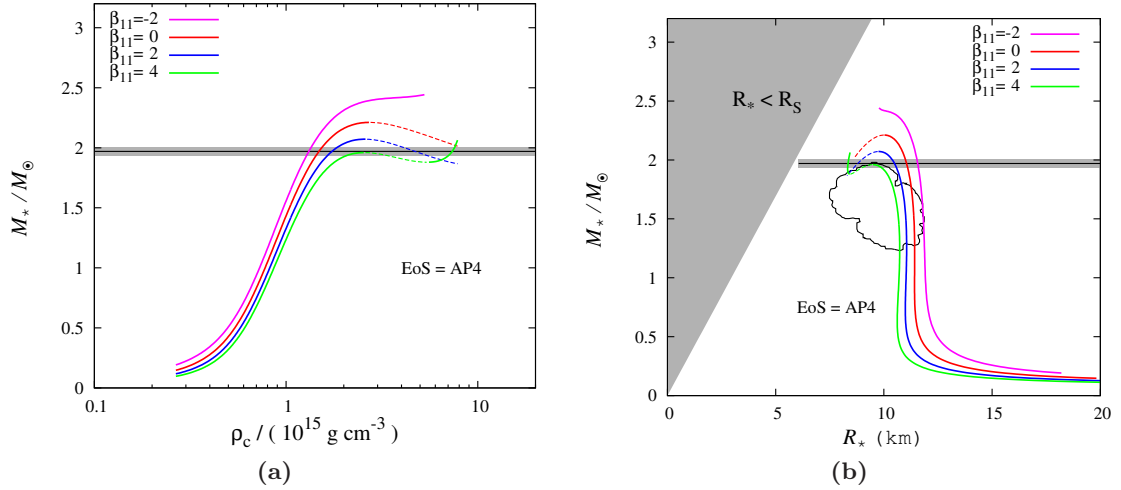
Again, we see that for  $\beta_{11} = -2$  (a value which we have already excluded by confronting with observations) a new stable solution branch exists for highest densities. For even lower values of  $\beta$  we obtain unstable ( $dm/dr < 0$ ) solutions for the highest densities. This is why we can not extend the solution to  $\beta_{11} = -3$  for densities greater than  $5 \times 10^{15} \text{ g cm}^{-3}$ .

### 3.5.3 AP4

The results for AP4 are shown in figure 3. It is seen from the M-R relation that for  $\beta_{11} > -2$ , AP4 is consistent with the confidence contours of [30] while the maximum mass becomes lower than  $\sim 2M_\odot$  for  $\beta_{11} > 4$  and is not compatible with the existence of PSR J1614-2230 [31]. Thus we conclude that  $-2 < \beta_{11} < 4$  for AP4.



**Figure 2.** The  $\rho_c - M$  (panel a) and M-R relation (panel b) for SLy EoS. See figure 1 for the notation in the figure. The results are discussed in section 3.5.2.

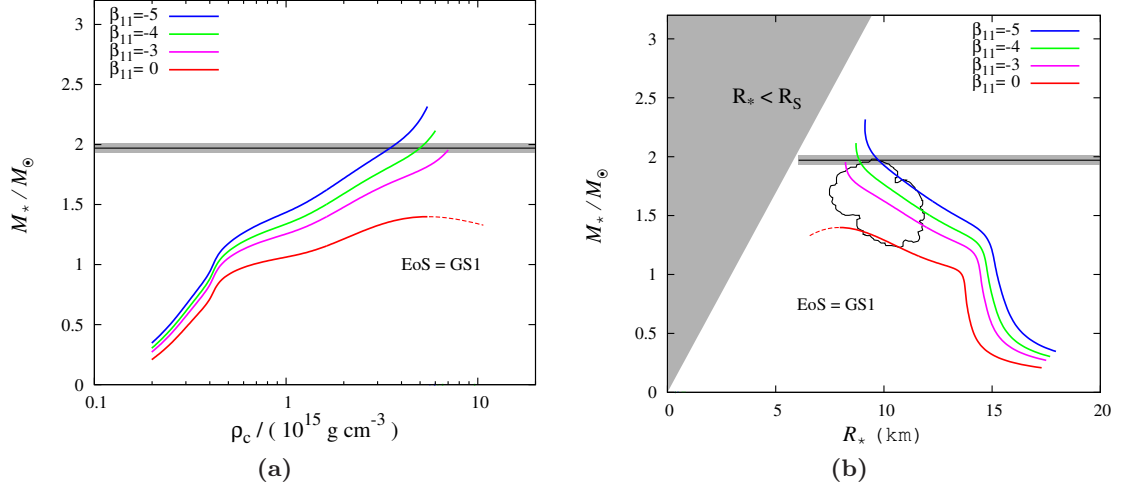


**Figure 3.** The  $\rho_c - M$  (panel a) and M-R relation (panel b) for AP4 EoS. See figure 1 for the notation in the figure. The results are discussed in section 3.5.3.

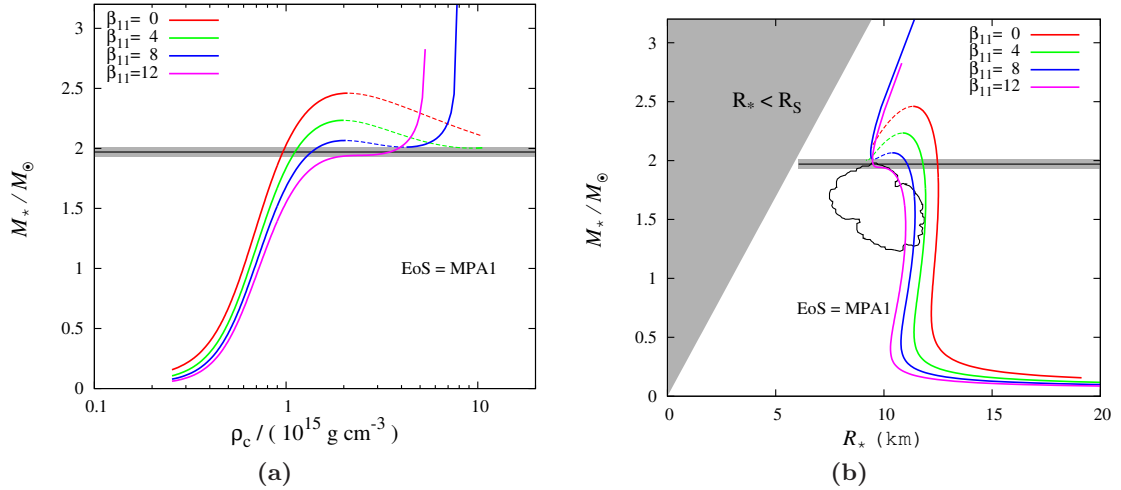
### 3.5.4 GS1

The mass versus the central density,  $\rho_c - M$ , and M-R relation for GS1 are shown in figure 4. GS1 is excluded within GR as its maximum mass is well below two solar masses. The maximum mass reaches  $\sim 2 M_\odot$  for  $\beta_{11} < -3$ . For  $\beta_{11} < -5$ , however, the M-R relation does not pass through the confidence contours of [30]. The observations then imply the constraint  $-5 < \beta_{11} < -3$  for GS1.

For negative values of  $\beta_{11}$  we find that the stability condition,  $dM/d\rho_c > 0$ , is satisfied for the whole range of central densities considered. As no maximum mass is reached, this situation that does not have a counterpart in general relativity.



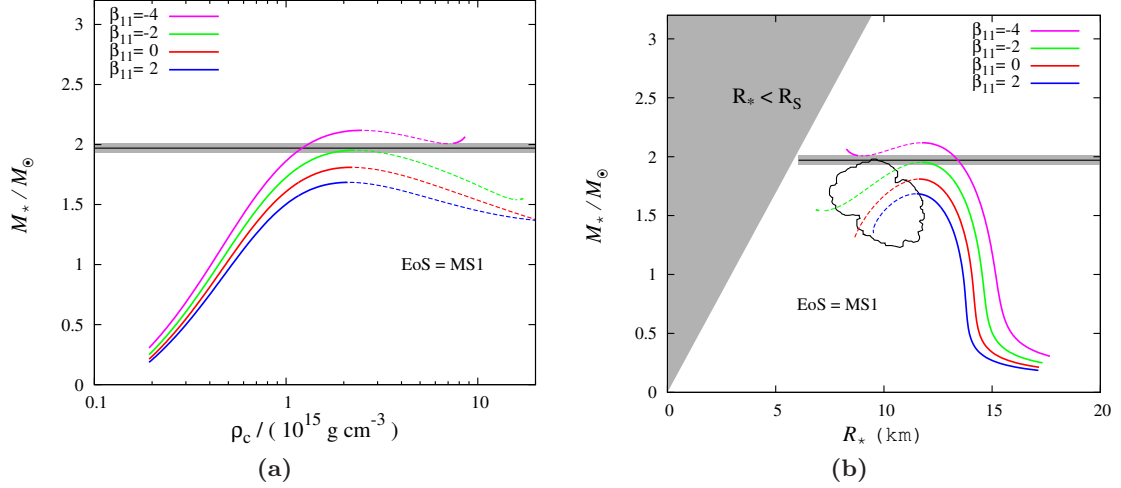
**Figure 4.** The  $\rho_c - M$  (panel a) and M-R relation (panel b) for GS1 EoS. See figure 1 for the notation in the figure. The results are discussed in section 3.5.4.



**Figure 5.** The  $\rho_c - M$  (panel a) and M-R relation (panel b) for MPA1 EoS. See figure 1 for the notation in the figure. The results are discussed in section 3.5.4.

### 3.5.5 MPA1

The relation between the central density and the mass of the NS,  $\rho_c - M$ , and M-R relation are shown in figure 5. The maximum mass of MPA1 is above the observed mass of PSR J1614-2230 for  $\beta_{11} < 12$ . For  $\beta_{11} < 4$  the M-R relation does not pass through the confidence interval of [30]. This then implies  $4 < \beta_{11} < 12$  for MPA1. For larger values of  $\beta$  the maximum mass obtained decreases as is the case for all EoSs. For  $\beta_{11} = 8$  we see that, apart from the usual stable (in the sense that  $dM/d\rho_c > 0$ ) and unstable branches, there emerges a stable branch for the highest central pressures which does not have a counterpart in GR. The unstable branch between the two stable branches disappears for even larger values of  $\beta$ . We thus observe that for  $\beta_{11} = 12$  we can obtain stable configurations with mass exceeding  $2M_\odot$ .



**Figure 6.** The  $\rho_c - M$  (panel a) and M-R relation (panel b) for MS1 EoS. See figure 1 for the notation in the figure. The results are discussed in section 3.5.5.

### 3.5.6 MS1

The  $\rho_c - M$  and M-R relation are shown in figure 6. This EoS has a maximum mass less than  $2M_\odot$  in GR and does not pass through the confidence contour presented in [30]. In order that the M-R relation has a maximum beyond the  $2M_\odot$  measured mass one needs  $\beta_{11} < -2$  which would take the M-R relation even further from the confidence contour of [30]. In order that the M-R relation marginally pass through the confidence contours of [30] one needs  $\beta_{11} > 2$  for which the maximum mass obtained is even smaller. We conclude from this analysis that MS1 is excluded as a possible EoS for NS in this gravity model assuming there is no systematic error in the M-R measurements of [30].

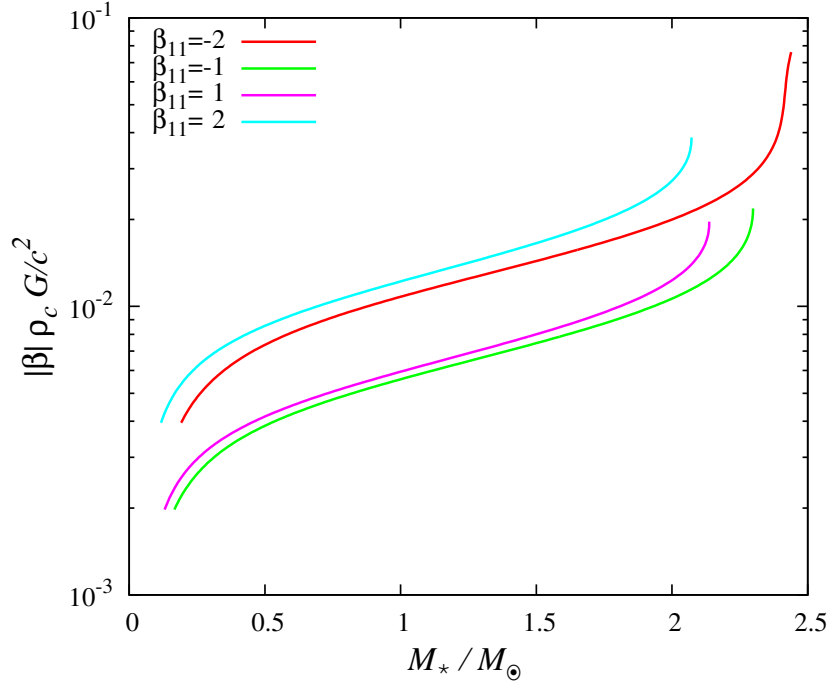
### 3.6 Validity of the perturbative approach

The validity of the perturbative method is justified if the terms that result from the  $\beta R_{\mu\nu}R^{\mu\nu}$  term in the Lagrangian are smaller than the terms arising from  $R$ , the GR term. This requires that  $|\beta|\rho_c \ll c^2/G$  be satisfied for each configuration we consider. In figure 7 we check this for EoS AP4 for a range of  $\beta$  values. We choose AP4 because it accommodates a large mass that could lead to the breakdown of the perturbative approach easily. We see that for large masses the dimensionless parameter that measures the perturbative term,  $|\beta|\rho_c G/c^2$ , reaches 0.1 which is critical in deciding whether the perturbative approach is justified. This indicates that the constraints we provide for  $\beta$  may not be accurate for masses approaching  $2M_\odot$  for  $|\beta| \sim 10^{11} \text{ cm}^2$  for any EoS.

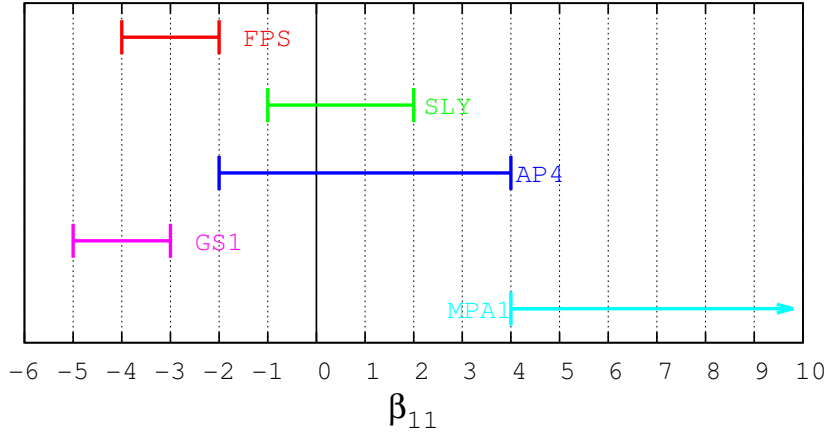
We observe from the figures in general that variations in the M-R relation comparable to employing different EoSs can be obtained for  $|\beta| \sim 10^{11} \text{ cm}^2$ . Using  $\beta \lesssim 10^{11} \text{ cm}^2$  gives M-R relations that can not be distinguished from the GR results. The range of  $\beta$  that is consistent with the observations are shown in figure 8 summarizing the results above. We see that  $|\beta_{11}| > 4$  is excluded for all EoSs we considered except for MPA1.

### 3.7 Dependence of maximum mass on $\beta$

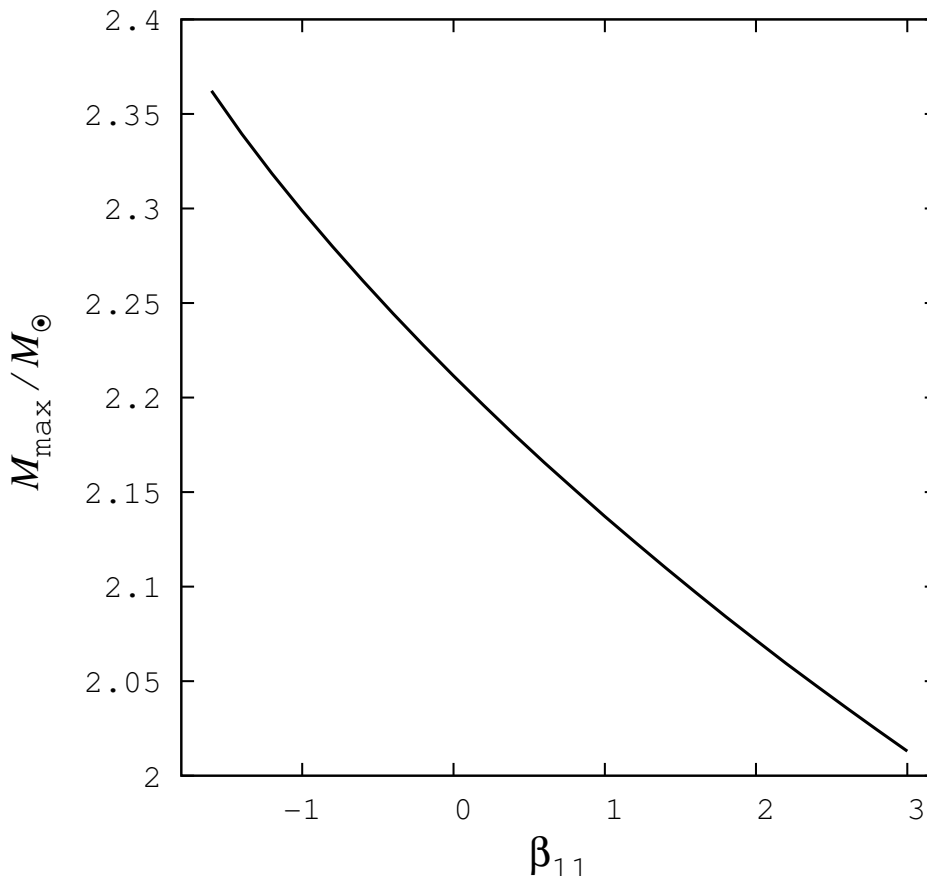
For all EoSs we observe that the maximum stable mass of a neutron star,  $M_{\text{max}}$ , and its radius at this mass,  $R_{\text{min}}$ , increases for decreasing values of  $\beta$ , for the ranges we consider



**Figure 7.** The dependence of the dimensionless perturbation parameter  $|\beta|\rho_c G/c^2$  on mass. The result is obtained for the AP4 EoS. We see that the validity of the perturbative approach is questionable for large masses and large values of  $\beta$ . This leads us to conclude that the constraints we provide for each EoS may not be as accurate, since the dimensionless perturbation parameter reaching 0.1 signals the break-down of the validity of the perturbative approach.



**Figure 8.** The range of  $\beta_{11} = \beta/(10^{11} \text{ cm}^2)$  consistent with the observations. MS1 is excluded from the plot, because it can not be reconciled with the confidence contours of [30] for the gravity model considered though it predicts NS masses well above the measured mass of PSR J1614-2230 [31] for  $\beta_{11} < -2$ . Note that  $|\beta_{11}| > 5$  is excluded for all EoSs we considered except for MPA1. The validity domain of MPA1 extends to very large positive values of  $\beta$ . We can not include its whole range as the perturbative approach applied in this work would break down for such large values.



**Figure 9.**  $M_{\text{max}}$  changing with  $\beta$  for the AP4 EoS. Note that for  $\beta_{11} < -1.8$ , in this gravity model, there is no maximum mass but only stable configurations.

in the figures. There is no change in the behavior of  $M_{\text{max}}$  and  $R_{\text{min}}$  values as  $\beta$  changes sign. Thus the structure of neutron stars in GR ( $\beta = 0$ ) does not constitute an extremal configuration in terms of  $M_{\text{max}}$  and  $R_{\text{min}}$ .

In figure 9 we show the dependence of these quantities on the value of  $\beta$  for the EoS AP4. We see that  $\beta = 0$  is not special point in figure.

## 4 Conclusions

In this paper we studied the structure of neutron stars (NSs) in a generalized theory of gravity motivated by string theory. In the first section we discussed the motivations for modifying gravity. In the second section we derived the field equations of this gravity model from its action. In the third section we obtained the hydrostatic equilibrium equations in spherical symmetry from the field equations by using a perturbative approach in which general relativity (GR) stands for the zeroth order gravity model. In the fourth section we solved the hydrostatic equilibrium equations for NSs by using numerical methods. In order to solve the equations we have used realistic equations of state (EoSs) that describe the dense matter inside NSs and obtained the mass-radius (M-R) relations depending on  $\beta$ , the free parameter



of the generalized gravity model considered. These M-R relations are then compared with the recent observational measurements of mass and radius of NSs to constrain the value of  $\beta$ .

We have shown that observationally significant changes on the M-R relation are induced for  $\beta \sim 10^{11} \text{ cm}^2$ . An order of magnitude smaller values for  $\beta$  gives results that are not significantly different from what GR predicts. An order of magnitude greater values, on the other hand, leads to results that can not be associated with known properties of NSs. For such values, the perturbative approach breaks down too. For this selection of EoSs we see that none of them are consistent with the observations for  $\beta < -5 \times 10^{11} \text{ cm}^2$  and  $\beta > 4 \times 10^{11} \text{ cm}^2$ . The only exception is MPA1 which accepts large positive values like  $\beta_{11} \sim 10$ . We thus conclude from this analysis that  $|\beta| \lesssim 5 \times 10^{11} \text{ cm}^2$  is an upper limit brought by observations of NSs [30, 31].

We note that, for the gravity model  $R + \alpha R^2$ , the constraint on  $\alpha$  obtained by [18] by using NSs confronting the M-R relation with the same observations, is of the order  $10^{10} \text{ cm}^2$ , two orders of magnitude smaller than the constraint obtained on  $\beta$  in the present work, although both  $\alpha$  and  $\beta$  have the same dimensions ( $[L]^2$ ). This indicates that the  $R^2$  term in eq. (1.2) is more effective on the structure of neutron stars than the  $R_{\mu\nu}R^{\mu\nu}$  term.

The last observation is very interesting from a theoretical point of view. As mentioned in the introduction, theory described by (1.2) is equivalent to Weyl tensor squared modification of general relativity for  $\beta = -3\alpha$  [24, 25]. That is, from the point of view of Weyl tensor squared modification,  $\alpha$  and  $\beta$  are in the same order. Our perturbative analysis in this paper also in some way forces us to make a similar statement: if  $\beta$  is taken in the same order as of  $\alpha$ , then there would not be the problem of the break-down of the perturbative approach. What extra we learn from this analysis is that even though the  $R^2$  and  $R_{\mu\nu}R^{\mu\nu}$  terms seem to be on equal footing in the expansion of a Weyl tensor square term,

$$\frac{1}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = -\frac{1}{3}R^2 + R_{\mu\nu}R^{\mu\nu} + \text{topological term}, \quad (4.1)$$

the contributions of them to neutron star physics are not equal. This is pointed out to us by the analysis in this paper and it is an important statement just by itself. It would be significant if this observation is confirmed and understood analytically. We plan to do this in a future publication.

We find that, some of the EoSs, which do not give M-R relations consistent with the observations within the framework of GR, can be reconciled with these observations via the free parameter  $\beta$  in the generalized gravity model considered in this work. This then brings up the question of degeneracy between the EoSs and the free parameter  $\beta$ . This degeneracy does not effect the constraint  $|\beta| \lesssim 5 \times 10^{11} \text{ cm}^2$  which is a bound for all EoSs we considered except for MPA1.

We finally comment that the constraint we obtained is actually the strongest constraint we could obtain by using NSs as the experimental apparatus. Another estimate of the nominal value  $\beta_0$  of the previous section is as follows: Typical radius of a NS is  $R_* \sim 10 \text{ km}$ , the only length scale in the system. This corresponds to an estimate of curvature  $R^{\mu\nu}R_{\mu\nu} \sim R_*^{-2} \sim 10^{-12} \text{ cm}^{-2}$  and so the new perturbative term  $\beta R^{\mu\nu}R_{\mu\nu}$  will become of order  $R_*$  and lead to variations on the structure of neutron stars for  $\beta \sim R_*^2 \sim 10^{12} \text{ cm}^2$ . As we obtain such variations in this limit, the value that we should obtain by using NSs, we infer that the actual value of  $\beta$  is likely much smaller than this. As we mentioned before, deviations from GR are not significant for NSs for values much less than this value.

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